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A PRESERVATION PRINCIPLE OF EXTREMAL MAPPINGS NEAR A STRONGLY PSEUDOCONVEX POINT AND ITS APPLICATIONS

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Introduction

Let D be a bounded domain in $\mathbf{C}^n$ and Δ the unit disc in $\mathbf{C}^1$. An extremal mapping (respectively, a complex geodesic) ϕ of D is a holomorphic mapping from Δ to D such that the Kobayashi metric of D at $\phi(0)$ and in the direction $\phi'(0)$ (respectively, the Kobayashi distance between any two points on $\phi(\Delta)$) is realized by ϕ. An obvious fact is that all complex geodesics are extremal.

In the one dimensional case, ϕ is extremal if and only if ϕ gives a covering mapping from Δ to D. In 1981, Lempert [Lm1] systematically studied the extremal mappings of a strongly convex domain. He proved that every extremal mapping of a C^k-strongly convex domain is actually a complex geodesic and admits a C^{k-2}-smooth extension up to the boundary ($k > 2$). As applications, he obtained the precise form of Fefferman's mapping extension theorem and the solutions of some types of Monge-Ampere equations [Lm1], [Lm2]. In [RW], some of Lempert's results were generalized to bounded convex domains. For non-convex domains, the abstract nature of the Kobayashi metric makes things more subtle. A simple investigation of the covering mappings of an annulus indicates immediately that (1) the extremal mappings may not be complex geodesics anymore and (2) the boundary behavior of extremal mappings may be very complicated although the domain is analytic and strongly pseudoconvex. In 1983, Poletskii [P] showed that the extremal mappings of a ρ-pseudoconvex domain must be almost proper and satisfy the Euler-Lagrange equations if the domain has in addition C^1 boundary. This result gives a very strong restriction for a holomorphic mapping to be extremal, and was later on used in some papers to characterize such mappings.

One purpose of this note is to present a preservation principle for extremal mappings near a C^3-strongly pseudoconvex point. Roughly speaking, we show that an extremal mapping with the initial point close to the bottom of a strongly pseudoconvex hole and with the initial velocity almost parallel to the

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