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CODIMENSION REDUCTION THEOREMS IN CONFORMAL GEOMETRY

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Introduction

We give the proof of the analogue, in conformal geometry, of the local version of the Theorem of Erbacher [E], and of theorems related to it.

The theorems obtained can also be viewed as the extension of classical theorems on reduction of codimension of a submanifold of a space of constant curvature to the case of a submanifold of a space locally conformal to a space of constant curvature that is locally conformally flat.

We start from the investigation of the geometric meaning of the nullity of the Willmore conformal forms, $\overset{r}{w}_M$, of a submanifold M. These conformal forms, introduced in [R], are invariant under conformal changes of the metric of the ambient space $\overline{M}$.

We prove:

THEOREM. *Let $\overline{M}$ be a locally conformally flat manifold. If M is a submanifold of $\overline{M}$ conformally nicely curved in $\overline{M}$, then $\overset{r}{w}_M$ is zero if and only if M is locally contained in a totally unbilical submanifold of $\overline{M}$ of dimension* $p = \dim \overset{r+1}{\Omega}_x M$ ($\overset{r+1}{\Omega}_x M$, $(r+1)$*-conformal osculating space*).

From the theorem, we deduce the local conformal version of the Erbacher Theorem (recently proved by Okomura [O] in the case of $\overline{M}$ of constant curvature).

COROLLARY III. *Let $\overline{M}$ be locally conformally flat and M a submanifold of $\overline{M}$. If $\overset{1}{W}_x M$ has constant dimension and is parallel in the normal bundle of M, then M is locally contained in a totally umbilical submanifold of $\overline{M}$ of dimension* $p = \dim M + \dim \overset{1}{W}_x M$ ($\overset{1}{W}_x M$ *first Willmore space*).

Introducing the notion of *conformally parallel distribution along a submanifold*, the analogue in conformal geometry of the notion of parallel distribution along a submanifold in Riemannian geometry, we are able to prove the

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