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PETTIS INTEGRALS AND SINGULAR INTEGRAL OPERATORS

BRIAN JEFFERIES AND SUSUMU OKADA

Introduction

The present note is concerned with conditions guaranteeing the integrability of operator valued functions acting on spaces $L^p(\mathbf{R}^n)$ for $1 < p < \infty$. To set the stage, some definitions and notation need to be fixed. Let E be a locally convex space. Let $(\Omega, \mathscr{S}, \mu)$ be a measure space. A scalarly μ-measurable function $\Psi\colon \Omega \to E$ is said to be *Pettis μ-integrable* if

$$\int_\Omega |\langle \Psi(\omega), \xi\rangle| \, d\mu(\omega) < \infty,$$

for every $\xi \in E'$, and for every $A \in \mathscr{S}$, there exists a vector $\Psi\mu(A) \in E$ such that $\langle \Psi\mu(A), \xi\rangle = \int_A \langle \Psi(\omega), \xi\rangle \, d\mu(\omega)$ for all $\xi \in E'$. In the context of Banach spaces, the notion of a vector valued function being integrable in this sense is, perhaps, less widely used than the familiar notion of Bochner integrability. If X is a Banach space with norm $\|\cdot\|$, then a function $\Psi\colon \Omega \to X$ is said to be *strongly μ-measurable* if it is the limit μ-a.e. of a sequence of X-valued $\mathscr{S}$-simple functions. If Ψ is strongly μ-measurable, then the function $\|\Psi\|\colon \Omega \to [0,\infty)$ defined by $\|\Psi\|(\omega) = \|\Psi(\omega)\|$ for all $\omega \in \Omega$ is μ-measurable, and Ψ is said to be *Bochner μ-integrable* if $\int_\Omega \|\Psi\| \, d\mu < \infty$. In this case, there exist X-valued $\mathscr{S}$-simple functions s_m, $m = 1, 2, \ldots,$ such that $\lim_{m\to\infty} \int_\Omega \|\Psi - s_m\| \, d\mu = 0$, and so for each $A \in \mathscr{S}$, if $\Psi\mu(A) \in E$ is defined by

$$\Psi\mu(A) = \lim_{m\to\infty} \int_A s_m \, d\mu,$$

then

$$\langle \Psi\mu(A), \xi\rangle = \int_\Omega \langle \Psi(\omega), \xi\rangle \, d\mu(\omega)$$

for all $\xi \in X'$, as required for the definition of Pettis integrability.

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