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REGULAR ORBITS OF NILPOTENT SUBGROUPS OF SOLVABLE GROUPS

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1. Introduction

Suppose that G is a solvable group, $\mathbf{F}$ a field, V a faithful $\mathbf{F}G$-module, and $A \subseteq G$ a nilpotent subgroup of G. The action of G permutes the vectors of V, and it is natural to ask under what conditions on G, A, $\mathbf{F}$, and V, the subgroup A is guaranteed to have a regular orbit on V. This question has been studied extensively and, if $\mathbf{F}$ has characteristic relatively prime to the order of A, definitive results have been obtained by T. R. Berger [2–8], B. Hargraves [16] and others. (See [9] for an overview of known results.)

When char($\mathbf{F}$) divides $|A|$, the picture is much less clear. P. Hall and G. Higman obtained the first related result in their renowned paper *On the p-length of p-soluble groups and reduction theorems for Burnside's problem* [15]. They show there that if A is a cyclic p-group, R an extraspecial r-group for some prime $r \neq p$, $\mathbf{F}$ a field of characteristic p that is also a splitting field for R, and G a group of the form $G = AR$, with A acting irreducibly and faithfully on $R/Z(R)$, then $V|_A$ *always* has a regular orbit. Although it appears to be very special, the Hall-Higman configuration is extremely important because it turns up regularly in minimal structures associated with many theorems.

The noncoprime configuration has also been studied by A. Espuelas. In [11] he studied the case in which A is a p-group and p = char $\mathbf{F}$. He proves the following theorem:

THEOREM 1.1 (Espuelas [11], p. 4). *Let G be a solvable group with $\mathcal{O}_p(G) = 1$ and let A be a p-subgroup of G, p a prime. Suppose that V is a faithful $\mathbf{F}G$-module with* char($\mathbf{F}$) $= p$. *If $p = 2$, assume that A is $Z_2 \wr Z_2$-free. Then V contains a regular A-orbit.*

In this paper, Espuelas asks whether his theorem can be extended to A nilpotent. It is the main purpose of this paper to extend Theorem 1.1 to allow

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