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CONJUGATE EXPANSIONS FOR HERMITE FUNCTIONS[1]

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1. Introduction

In the last chapter of [St], Stein discusses the notion of a Hilbert transform associated with a general Sturm-Liouville operator. In particular, let

$$L = \frac{d^2}{dx^2} + a(x)\frac{d}{dx}$$

with $a'(x) \leq 0$ and let

$$q(x) = \exp\left(\int_0^x a(t)\,dt\right).$$

Let $\{\varphi_n\}$, $n \geq 0$, be a complete orthonormal set of eigenfunctions of L with eigenvalue $-\lambda_n^2$ for the Hilbert space $L^2(\mathbf{R}, q(x)\,dx)$. Then Stein points out that

$$\left\{\frac{1}{\lambda_n}\frac{d\varphi_n}{dx}\right\}$$

is also an orthonormal system for $L^2(\mathbf{R}, q(x)\,dx)$ and the suggested Hilbert transform is given by the mapping

$$\varphi_n \to \frac{1}{\lambda_n}\frac{d\varphi_n}{dx},\ n \geq 0.$$

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[1] *Added on November 28, 1993.* The authors have recently become aware of the paper by I. Joó, *On Hermite-Fourier series,* Periodica Math. Hung. **24**(1992), 87-118, where another notion of conjugacy for Hermite function expansions is defined and investigated. Specifically the formal Hilbert transform that emerges is given by the mapping $h_n \to h_{n-1}$. The statements of main results from our paper and that of Joó though identical deal with different objects. Also the proofs in each case which are modelled on Muckenhoupt's technique are similar.

[2] The paper was written while the second author was visiting the Department of Mathematics, University of Georgia, during the 1990-1991 academic year.