

FULL NEST ALGEBRAS

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I. Introduction

Direct limits of finite dimensional operator algebras have become a source for many interesting examples of non-self-adjoint operator algebras. This note will study a special class of such algebras—direct limits of full upper triangular matrix algebras with nest preserving embeddings. These limit algebras are members of three important classes of operator algebras: they are simultaneously nest subalgebras, analytic subalgebras, and strongly maximal triangular subalgebras of the UHF C^* -algebras which they generate.

In what follows, we shall primarily study directed systems with the following form:

$$T_{n_1} \xrightarrow{\nu_1} T_{n_2} \xrightarrow{\nu_2} T_{n_3} \xrightarrow{\nu_3} \cdots \longrightarrow A$$

where T_n is the algebra of upper triangular $n \times n$ matrices and each ν is a unital isometric homomorphism. We further require that each embedding ν satisfy the following properties:

- (1) ν has an extension to a $*$ -homomorphism of M_n .
- (2) ν maps a matrix unit in T_{n_k} to a sum of matrix units in $T_{n_{k+1}}$.
- (3) ν maps $\mathcal{L}at T_{n_k}$ into $\mathcal{L}at T_{n_{k+1}}$.

Properties 1 and 2 are standard assumptions (but see [P3, P4] for information on what may happen if these assumptions are not satisfied). Property 3 is satisfied by the refinement embedding, $\rho: T_n \rightarrow T_{nk}$, which is defined as follows: $\rho[a_{ij}] = [a_{ij}I_k]$, for all $[a_{ij}] \in T_n$. The embedding most often contrasted with the refinement embedding, the standard embedding, $\sigma: T_n \rightarrow T_{nk}$, defined by $\sigma(a) = a \oplus \cdots \oplus a$ (k factors), does not satisfy property 3.

DEFINITION 1.1. An embedding ν which satisfies properties 1, 2, and 3 will be called a *nest embedding*.

Received July 22, 1992.

1991 Mathematics Subject Classification. Primary 47D25; Secondary 46H35, 46L89.

¹The authors would like to thank C. Laurie, B. Wagner and Y.T. Poon for helpful conversations on the subject matter of this paper.