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FORMALIZABILITY OF DG MODULES AND MORPHISMS OF CDG ALGEBRAS

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The aim of this paper is to study homotopy types, more precisely, formality, of certain local systems over compact Kähler manifolds, following works of J. Morgan [Mor] and V. Navarro Aznar [Na_1], [Na_2]. Global sections of these local systems can be seen as dg modules over cdg algebras. Following [Sul], we prove that formalizability of such dg modules does not depend on the ground field (Theorem 2.2). Results and proofs are easily translated for cdg algebra morphisms, so we develope the case of dg modules in detail and confine ourselves to state them for cdg algebra morphisms, remarking differences whenever they can arise. For both situations, our principal tool is the minimal model.

D. Sullivan's theory of the minimal model says that, for a rational space X, the $\mathbf{Q}$-homotopy type is determined by a minimal model of the $\mathbf{Q}$-cdg algebra $A_{PL}(X)$ (see [Sul]). For certain spaces, this minimal model, and so its homotopy type, is a formal consequence of its rational cohomology algebra $H^*(X;\mathbf{Q})$. They are called *formal* spaces. Among them one can find Lie groups, classifying spaces, compact Kähler manifolds... Formality of these latter ones was proved in [D-G-M-S] over the real numbers. The descent of formality from $\mathbf{R}$ to $\mathbf{Q}$ is proved in [Sul]. This is done in two steps: first, one gives a caracterization of formality in terms of the lifting property of automorphism from the cohomology algebra to the algebra [Sul, Theorem 12.7]. Second, one sees that this property does not depend on the ground field [Sul, Theorem 12.1]. For morphisms of cdg algebras, the first one of these results is due to Y. Felix and D. Tanré (see [F-T]). The second one can be found in [ViP_1]. In this paper, we give independent proofs of both results, which, in the line of Sullivan's, do not depend on the choice of a particular construction of models with extra structure (namely, filtrations), but rather on abstract properties of formalizability and minimality (Theorems 3.1 and 3.2).

The paper is organized as follows: in § 1, we give some preliminaries which will allow us to translate Sullivan's results from cdg algebras to dg modules and cdg algebra morphisms. Particularly, we define the notions of formalizability and minimality in abstract terms in such a way that they include the cases of cdg algebras, dg modules over cdg algebras, morphisms of cdg

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