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THE MULTIPLIER OPERATORS ON THE PRODUCT SPACES

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Introduction

Let $H^p(R^{n_1} \times R^{n_2})$ be the Hardy space defined on the product spaces (for more details, see [1]) and let a function $a(x_1, x_2)$ denote a rectangle p atom on $H^p(R^{n_1} \times R^{n_2})$ if (i) the $a(x_1, x_2)$ is supported on a rectangle $R = I \times J$ (I and J are cubes on R^{n_1} and R^{n_2} respectively), (ii) $\|a\|_2 \le |R|^{1/2-1/p}$ and (iii) one picks and fixes two sufficiently large positive integers k and l (depending on p) such that

$$\int_I x_1^{\alpha} a(x_1, x_2)\, dx_1 = 0 \qquad \text{for all } x_2 \in J \text{ and } |\alpha| \le k$$

$$\int_J x_2^{\beta} a(x_1, x_2)\, dx_2 = 0 \qquad \text{for all } x_1 \in I \text{ and } |\beta| \le l.$$

In the paper [3], R. Fefferman gave a very powerful theorem (see Theorem 1) for studying the boundedness on the $H^p(R^{n_1} \times R^{n_2})$ spaces of a linear operator. In his theorem, it mentioned that to consider the boundedness on H^p of a linear operator one only needs to look at the boundedness of the linear operator acting on the rectangle p atoms. This is true despite the counterexample of L. Carleson which shows that the space $H^p(R^{n_1} \times R^{n_2})$ cannot be decomposed into rectangle atoms.

We will use $\wedge$ to denote the Fourier Transform and $\wedge_1$ to denote the Fourier Transform acting on the first variable. Throughout this paper, C represents a constant, although different in different places. T_m denotes the multiplier operator associated with the multiplier m, i.e.,

$$\widehat{T_m f}(\xi, \eta) = m(\xi, \eta) \hat{f}(\xi, \eta).$$

THEOREM 1 (R. Fefferman [3]). *Suppose that T is a bounded linear operator on $L^2(R^{n_1} \times R^{n_2})$. Suppose further that if a is an $H^p(R^{n_1} \times R^{n_2})$*

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