ILLINOIS JOURNAL OF MATHEMATICS
Volume 38, Number 3, Fall 1994

A NOTE ON UNCONDITIONAL STRUCTURES IN WEAK HILBERT SPACES

N. J. NIELSEN

Introduction

This note is to be considered as an addendum of [3], where an extensive study of Banach lattices, which are weak Hilbert spaces, was made. We prove that if a non-atomic separable Banach lattice is a weak Hilbert space then it is lattice isomorphic to $L_2(0,1)$. The result is a consequence of Theorem 3.11 in [3] together with an easy, short argument.

We believe that it may have some impact on the study of unconditional structures in weak Hilbert spaces.

For the convenience of the reader, in Section 1 we have given the definition of a weak Hilbert space and formulated the special case of [3], Theorem 3.11, which is needed to prove our result.

1. Notation and terminology

In this note we shall use the notation and terminology commonly used in Banach space theory as it appears in [1] and [2].

If E is a finite dimensional Banach space we denote the Banach-Mazur distance between E and the Hilbert space by $d(E)$, and if (x_j) is a sequence in a Banach space X we let $[x_j]$ denote the closed linear span of (x_j). Given a finite set A we let $|A|$ denote the cardinality of A. One of the many equivalent characterizations of a weak Hilbert space given by Pisier [4] we use as the definition.

DEFINITION 1. *A Banach space X is called a weak Hilbert space, if there exist a $\delta > 0$ and a $C \geq 1$ such that for every finite dimensional subspace $E \subseteq X$, there exist a subspace $F \subseteq E$ and a projection $P: X \to F$ with* $\dim F \geq \delta \dim E$, *$d(F) \leq C$ and $\|P\| \leq C$.*

The result below, which is a special case of one of the main theorems of [3], Theorem 3.11, is the main tool of this note.

Received May 24, 1992.
1991 Mathematics Subject Classification. Primary 46B20; Secondary 46B42.