ILLINOIS JOURNAL OF MATHEMATICS
Volume 38, Number 3, Fall 1994

KNOTS AND SHELLABLE CELL PARTITIONINGS OF S^3

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A *cell partitioning* of S^3 is a finite covering H of S^3 by 3-cells such that if m is any positive integer and exactly m 3-cells of H intersect, their common part is a cell of dimension $4 - m$, where cells of negative dimension are empty. The 3-cells of a cell partitioning of S^3 fit together in a staggered, brick-like pattern.

A cell partitioning H of S^3 is *shellable* if and only if there is a counting $\langle h_1, h_2, \cdots, h_n \rangle$ of H such that if i is an integer and $1 \leqq i < n$, then $h_1 \cup h_2 \cup \cdots \cup h_i$ is a 3-cell. Such a counting is a *shelling* of H.

In this paper, we shall study a connection between knots in S^3 and shellability of cell partitionings of S^3. We shall use these results to construct nonshellable cell partitionings of S^3.

Our results involve the use of the bridge number of a knot in S^3. In Section 1 of this paper, we shall review some results concerning knots in S^3 and bridge numbers of knots in S^3. In Section 2, we shall establish the main result of the paper. In Section 3, we shall establish a variant of the main result that is useful in some situations. In Section 4, we shall use the results of this paper to construct a nonshellable cell partitioning of S^3 and, as a variation on that construction, a nest of nonshellable cell partitionings of S^3.

Throughout this paper, we shall assume that S^3 has its standard piecewise linear structure.

The author thanks the referee for suggestions and corrections.

1. Knots in S^3

A *knot* in S^3 is a polygonal simple closed curve in S^3. Two knots k and l in S^3 are of the *same knot type* in S^3 if and only if there is an orientation preserving PL homeomorphism $f: S^3 \to S^3$ such that $f(k) = l$. A knot in S^3 is *trivial* if and only if it has the same knot type as the boundary of a 2-simplex in S^3.

Suppose C is a 3-cell. Then α is a *spanning arc* of C if and only if α is an arc in C such that $\operatorname{Bd} \alpha \subset \operatorname{Bd} C$ and $\operatorname{Int} \alpha \subset \operatorname{Int} C$. D is a *semispanning disc* of C if and only if D is a disc in C such that $\operatorname{Int} D \subset \operatorname{Int} C$ and $D \cap \operatorname{Bd} C$ is an arc on $\operatorname{Bd} C$. The statement that β is a *straight* spanning arc of C means that β is a spanning arc of C and there is a semispanning disc D of C such that $\beta \subset \operatorname{Bd} D$. Recall that if β is a polyhedral straight spanning arc of a

Received February 28, 1991.
1991 Mathematics Subject Classification. Primary 57M50; Secondary 57M25.