ILLINOIS JOURNAL OF MATHEMATICS
Volume 39, Number 1, Spring 1995

TANGENTIAL HARMONIC APPROXIMATION ON RELATIVELY CLOSED SETS

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1. Introduction

Let Ω be an open set in Euclidean space $\mathbb{R}^n$ $(n \geq 2)$ and E be a relatively closed subset of Ω. A subset A of Ω will be called *Ω-bounded* if its closure $\bar{A}$ is a compact subset of Ω. We use $\hat{E}$ to denote the union of E with all Ω-bounded (connected) components of $\Omega \setminus E$. As usual, A° and ∂A will denote respectively the interior and boundary of a set A. Also, $C(A)$ will denote the collection of all real-valued continuous functions on A, and $\mathscr{H}(A)$ (resp. $\mathscr{S}^+(A)$) will denote the collection of functions which are harmonic (resp. positive and superharmonic) on some open set containing A. We will say that the pair (Ω, E) *satisfies the (K, L)-condition* if, for each compact subset K of Ω, there is a compact subset L of Ω which contains every Ω-bounded component of $\Omega \setminus (E \cup K)$ whose closure intersects K. The Alexandroff compactification of Ω will be denoted by Ω^*. We note that $\Omega^* \setminus E$ is connected if and only if $\hat{E} = E$ and that, if this is the case, then $\Omega^* \setminus E$ is locally connected if and only if (Ω, E) satisfies the (K, L)-condition. The following result was recently established by Armitage and Goldstein [3, Theorem 1.1].

THEOREM A. *Let Ω be a connected open set in $\mathbb{R}^n$ which possesses a Green function $G(\cdot, \cdot)$, let E be a relatively closed subset of Ω and let $P \in \Omega$. If $\Omega^* \setminus E$ is connected and locally connected, then for each h in $\mathscr{H}(E)$ and for each positive number ε there exists H in $\mathscr{H}(\Omega)$ such that*

$$|H(X) - h(X)| < \varepsilon \min\{1, G(P, X)\} \quad (X \in E).$$

Using Theorem A and material from [9] we will prove the following. The reader is referred to Helms [12] or Doob [7] for an account of thin sets and the fine topology.

THEOREM 1. *Let Ω be a connected open set in $\mathbb{R}^n$ and E a relatively closed proper subset of Ω. The following are equivalent.*

(a) *For each h in $\mathscr{H}(E)$ and each positive number ε, there exists H in $\mathscr{H}(\Omega)$ such that $|H - h| < \varepsilon$ on E.*

Received October 30, 1992.
1991 Mathematics Subject Classification. Primary 31B05.