RIEMANNIAN SUBMERSIONS WHICH PRESERVE THE EIGENFORMS OF THE LAPLACIAN

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Let \(\pi: Z \rightarrow Y\) be a Riemannian submersion where \(Y\) and \(Z\) are closed Riemannian manifolds. Let \(E(\lambda, \Delta^Y_p) \subseteq C^\infty \Lambda^p Y\) and \(E(\lambda, \Delta^Z_p) \subseteq C^\infty \Lambda^p Z\) be the eigenspaces of the \(p\) form valued Laplacians on \(Y\) and on \(Z\). We say the pullback

\[
\pi^*: C^\infty \Lambda^p Y \rightarrow C^\infty \Lambda^p Z
\]

preserves the \(p\) eigenforms of the Laplacian if for any \(\lambda \in \mathbb{R}\), there exists \(\mu(\lambda) \in \mathbb{R}\) so that

\[
\pi^* E(\lambda, \Delta^Y_p) \subseteq E(\mu(\lambda), \Delta^Z_p);
\]

in other words \(\pi^* \Phi\) is an eigenform of \(\Delta^Z_p\), although with a possibly different eigenvalue, for every eigenform \(\Phi\) of \(\Delta^Y_p\).

**THEOREM 1.** The following conditions are equivalent:

(a) The fibers of \(\pi\) are minimal submanifolds.
(b) \(\Delta^Z_0 \pi^* = \pi^* \Delta^Y_0\).
(c) \(\pi^*\) preserves the eigenfunctions of the Laplacian \(\Delta^Y_0\).

**THEOREM 2.** The following conditions are equivalent:

(a) The fibers of \(\pi\) are minimal submanifolds and the horizontal distribution of \(\pi\) is integrable.
(b) For all \(0 \leq p \leq \text{dim}(Y)\), \(\Delta^Z_p \pi^* = \pi^* \Delta^Y_p\).
(c) There exists \(p\) with \(1 \leq p \leq \text{dim}(Y)\) such that \(\pi^*\) preserves the \(p\) eigenforms of the Laplacian \(\Delta^Y_p\).

These results deal with the totality of the eigenspaces; the following result deals with a single eigenform.

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