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ORTHOGONAL MARTINGALES UNDER DIFFERENTIAL SUBORDINATION AND APPLICATIONS TO RIESZ TRANSFORMS

RODRIGO BAÑUELOS AND GANG WANG

1. Introduction

Let $\mathbb{H}$ be a separable Hilbert space with norm $|\cdot|$ and inner product $\langle\cdot,\cdot\rangle$. Let $\mathcal{F} = \{\mathcal{F}_t\}_{t\geq 0}$ be a family of right-continuous sub-σ-fields of a probability space $\{\Omega, P, \mathcal{A}\}$ such that $\mathcal{F}_0$ contains all sets of probabilty zero. For two $\mathcal{F}$-adapted continuous-time $\mathbb{H}$-valued martingales $X = \{X_t\}_{t\geq 0}$ and $Y = \{Y_t\}_{t\geq 0}$, let $[X, Y] = \{[X, Y]_t\}_{t\geq 0}$ be the quadratic covariation process between X and Y (see, for example, [DM]). Unless otherwise stated, we assume all the martingales in the paper are $\mathbb{H}$-valued where $\mathbb{H}$ is a separable Hilbert space, and have right-continuous paths with left-limits (r.c.l.l.). For notational simplicity, we use $[X] = \{[X]_t\}_{t\geq 0}$ to denote $[X, X]$. We say the martingale Y is *differentially subordinate to* the martingale X, if $[X]_t - [Y]_t$ is a nondecreasing and nonnegative function of t. The notion of differential subordination permits generalizations of various sharp martingale inequalities of Burkholder [Bur 1-4] from the discrete-time and diverse stochastic integral settings to the present more general setting (see [Wan]). For example, if X and Y are continuous-time martingales with Y being differentially subordinate to X, then Theorem 1 of [Wan] says

$$\|Y\|_p \leq (p^* - 1)\|X\|_p, \qquad \text{for } 1 < p < \infty \tag{1.1}$$

where $p^* = \max\{p, p/(p-1)\}$ and the inequality is strict if $p \neq 2$ and $0 < \|X\|_p < \infty$. It is also sharp since it is already sharp in the special cases considered in [Bur1]. For a martingale X, the norm $\|X\|_p$ is defined by

$$\|X\|_p = \sup_t \|X_t\|_p.$$

Because of the close relationship between martingales and harmonic analysis, new sharp inequalities under differential subordination for continuous-time martingales have very important applications in analysis. For example, in Bañuelos and Wang

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