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COMPOSITION OPERATORS ON SMALL WEIGHTED HARDY SPACES

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1. Introduction

Let φ be an analytic map of the unit disk D into itself and define $C_\varphi(f) = f \circ \varphi$ whenever f is analytic on D. We are interested here in studying basic properties (e.g., boundedness, compactness) of the composition operator C_φ acting on weighted Hardy spaces $H^2(\beta)$, defined from a weight sequence $\{\beta(n)\}_0^\infty$ satisfying $\beta(0) = 1$, $\beta(n) > 0$ and $\lim_{n\to\infty} \beta(n)^{\frac{1}{n}} = 1$. Given such a sequence, $f(z) = \sum_{n=0}^\infty a_n z^n$ is in $H^2(\beta)$ if and only if

$$\|f\|_\beta^2 = \sum_{n=0}^\infty |a_n|^2 \beta(n)^2 < \infty.$$

Note that $H^2(\beta)$ will be a Hilbert space of analytic functions on D with inner product

$$\langle \sum_{n=0}^\infty a_n z^n, \sum_{n=0}^\infty c_n z^n \rangle_\beta = \sum_{n=0}^\infty a_n \bar{c}_n \beta(n)^2,$$

for which the monomials $\{z^n\}_0^\infty$ form a complete set of non-zero orthogonal vectors.

The terminology "weighted Hardy space" comes of course from the observation that if $\beta(n) \equiv 1$, then $H^2(\beta)$ is the usual Hardy Hilbert space $H^2(D)$. For other particular choices of $\{\beta(n)\}_0^\infty$, the corresponding space $H^2(\beta)$ may turn out to be a familiar space; we will note these as the occasion arises. If $\{\beta_1(n)\}$ and $\{\beta_2(n)\}$ are two weight sequences with

$$\frac{1}{c}\,\beta_2(n) \le \beta_1(n) \le c\,\beta_2(n) \ \text{ for some } c \in (0, +\infty),$$

then $H^2(\beta_1) = H^2(\beta_2)$, with equivalent norms.

Our principal interest in this paper will be with "small" weighted Hardy spaces. The precise meaning of small will vary somewhat from theorem to theorem. At the very least we will require that

$$\sum_{n=0}^\infty \frac{1}{\beta(n)^2} < \infty,$$

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