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A HECKE CORRESPONDENCE THEOREM FOR MODULAR INTEGRALS WITH RATIONAL PERIOD FUNCTIONS

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1. Introduction

In the 1930's Erich Hecke used the Mellin transform and its inverse to demonstrate a systematic relationship between automorphic forms and Dirichlet series [5], [6]. In particular, entire modular forms on the full modular group $\Gamma(1) = SL(2, \mathbf{Z})$ correspond to Dirichlet series which satisfy a functional equation.

In [2], Eichler introduced generalized abelian integrals which he obtained by integrating modular forms of positive weight. An Eichler integral satisfies a modular relation with a polynomial period function. In [8] and [9], Marvin Knopp generalized Eichler integrals and developed the theory of modular integrals with rational period functions.

In [9], Knopp shows that an entire modular integral with a rational period function corresponds to a Dirichlet series which satisfies Hecke's functional equation, provided the rational period function has poles only at 0 or ∞. Knopp also proves a converse theorem, from which it follows that if the rational period function has any other poles the corresponding Dirichlet series does not satisfy the same functional equation.

In [4], Hawkins and Knopp prove a Hecke correspondence theorem in which a modular integral with an arbitrary rational period function corresponds to a Dirichlet series which satisfies a more general functional equation. In this case the functional equation for the Dirichlet series contains an additional remainder term which arises from the poles of the rational period function which are not at 0 or ∞. Hawkins and Knopp formulate their results for modular integrals on the theta group, Γ_θ, a subgroup of index 3 in $\Gamma(1)$. The theta group has a single group relation and any rational period function on Γ_θ must satisfy a corresponding relation. This relation in turn imposes a relation on the remainder term in the functional equation for the corresponding Dirichlet series.

In this paper we present a Hecke correspondence theorem for modular integrals of weight $2k \in 2\mathbf{Z}^+$ with rational period functions on the *full* modular group $\Gamma(1)$. The modular group has a second group relation which imposes more structure (than Γ_θ) on any modular integral, forcing its rational period function to satisfy a second relation. This in turn imposes more structure on the remainder term in the functional equation for the corresponding Dirichlet series. We will modify the characterization of rational

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