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CONFORMAL MOTION OF CONTACT MANIFOLDS WITH CHARACTERISTIC VECTOR FIELD IN THE k-NULLITY DISTRIBUTION

RAMESH SHARMA AND DAVID E. BLAIR

Dedicated to the memory of Professor Kentaro Yano

1. Introduction

It is known (see for example, [17]) that if an m-dimensional Riemannian manifold admits a maximal, i.e., an $(m+1)(m+2)/2$-parameter group of conformal motions, then it is conformally flat. It is also known [9] that a conformally flat Sasakian (normal contact metric) manifold is of constant curvature 1. This shows that the existence of maximal conformal group places a severe restriction on the Sasakian manifold. Thus one is led to examine the effect of the existence of a single 1-parameter group of conformal motions on a Sasakian manifold. All the transformations considered in this paper are infinitesimal. Okumura [10] proved that a non-isometric conformal motion of a Sasakian manifold M of dimension $2n+1$ $(n > 1)$ is special concircular and hence if, in addition, M is complete and connected then it is isometric to a unit sphere. The proof is based on Obata's theorem [8]: "Let M be a complete connected Riemannian manifold of dimension $m > 1$. In order for M to admit a non-trivial solution ρ of the system of partial differential equations $\nabla\nabla\rho = -c^2\rho g$ (c = a constant > 0), it is necessary and sufficient that M be isometric to a unit sphere of radius $1/c$." The purpose of this paper is (i) to extend Okumura's result to dimension 3 and (ii) to study conformal motion of the more general class of contact metric manifolds (M, η, ξ, ϕ, g) satisfying the condition that the characteristic vector field ξ belongs to the k-nullity distribution $N(k)$: $p \to N_p(k) = \{Z \text{ in } T_pM\colon R(X, Y)Z = k(g(Y, Z)X - g(X, Z)Y)$ for any X, Y in T_pM and a real number $k\}$ (see Tanno [15]). For $k = 1$, M is Sasakian. For $k = 0$, M is flat in dimension 3 and in dimension $2n+1 > 3$, it is locally the Riemannian product $E^{n+1} \times S^n(4)$ (see Blair [3]). We say that a vector field v on M is an infinitesimal contact transformation [12] if $£_v\eta = f\eta$ for some function f where £ denotes the Lie-derivative operator. We also say that a vector field v on M is an automorphism of the contact metric structure if v leaves all the structure tensors η, ξ, ϕ, g invariant (see [13]).

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