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NORM INEQUALITIES FOR VECTOR VALUED RANDOM SERIES

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1. Introduction

It is well known that Rademacher functions, $r_n(t)$, which are defined by

$$r_0(t) = \begin{cases} 1 & 0 \le t < \frac{1}{2} \\ -1 & \frac{1}{2} \le t < 1 \end{cases}, \qquad r_0(t+1) = r_0(t), \quad r_n(t) = r_0(2^n t),\ n \ge 1,$$

form a sequence of independent, symmetric and identically distributed random variables. Rademacher series $\sum r_j(t)u_j$ where u_j belong to a Banach space have been investigated extensively; see [1], [5], [9], [12], [13].

An important result for Rademacher series is the Khinchin-Kahane inequality: for any $0 < q < p < \infty$, there exists constant $b(p, q)$ such that for any $N > 1$,

$$\left\| \sum_{j=1}^{N} r_{j-1}u_j \right\|_p \le b(p,q) \left\| \sum_{j=1}^{N} r_{j-1}u_j \right\|_q$$

holds in any Banach space.

This inequality holds for a large class of zero–mean random variables; see [2], [4], [5], [8], [14]. We extend the inequality to a class of nonzero-mean random variables and we show that a constant vector can be added to both sides of the inequality. The latter enables us to study vector valued versions of the Burkholder local distribution estimates which Stein used in the proof of his celebrated theorem on limits of sequences of operators [13]. In a subsequent paper we will give a vector valued version of Stein's theorem [13] by using this local distribution estimate.

We prove vector valued local norm inequalities in L^p as well as in some Orlicz spaces for certain independent random variables which satisfy the Khinchin-Kahane inequality. We show that the local behavior of the tail series is the same as the global behavior of the series itself.

2. An extension of the Khinchin-Kahane inequality

Throughout the paper, for a sequence of independent random variables $\{X_j\}$, we will denote $X = \{X_j\}$.

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