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L_2 COHOMOLOGY OF THE BERGMAN METRIC FOR WEAKLY PSEUDOCONVEX DOMAINS

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1. Introduction

Suppose that Ω is a bounded pseudoconvex domain in C^n. The Bergman metric of Ω is a naturally defined Kaehler metric. We restrict our attention to those Ω whose Bergman metric is complete. By a theorem of Ohsawa [12], this includes all pseudoconvex domains with C^1 boundary. Every biholomorphic automorphism of Ω induces an isometry in the Bergman metric. The Hopf-Rinow theorem therefore implies that the Bergman metric of a homogeneous domain is complete.

Let $H_2^i(\Omega)$ denote the space of square integrable harmonic i-forms relative to the Bergman metric. The following result was proved in 1983 [6]:

THEOREM 1.1. *If Ω is strictly pseudoconvex, then $H_2^i(\Omega) = 0$, for $i \neq n$.*

Ohsawa and Takegoshi developed this work by giving both alternative proofs of Theorem 1.1 and applications to extension problems in several complex variables [13], [14]. Ideas of Gromov [8], were applied in [5] to give a conceptually clear proof of the results from [6].

In [5] the author developed a result of Gromov to give a criterion on the Bergman metric for the vanishing of $H_2^i(\Omega)$ for $i \neq n$ on domains in C^n for which the Bergman metric is complete. This criterion is the existence of a positive constant c_2 such that the estimate in Proposition 2.3 holds for all non-zero tangent vectors at all points. This approach led to a simpler proof of the earlier result in [6] for strictly pseudoconvex domains.

In the present paper we investigate this criterion more generally. We prove that it holds for pseudoconvex domains of finite type in C^2 and for locally convexifiable domains of finite type in C^n. We also verify it for homogeneous domains and for the domains with large automorphism groups given by $|z|^2 + |w|^{2p} < 1$, $p > 1$, in all dimensions. We provide an example of a bounded pseudoconvex Reinhardt domain in C^3 for which the criterion fails. The defining equation is $|w|^2 + |z_1 z_2|^2 + |z_1|^{10} + |z_2|^{10} < 1$. We do not determine however whether the cohomology vanishes for this domain.

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