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# PARITY OF FOURIER COEFFICIENTS OF MODULAR FORMS

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## 1. Introduction

A partition of a non-negative integer $n$ is a non-increasing sequence of positive integers whose sum is $n$. It is of interest to examine the number of partitions of $n$ under some additional restriction on the summands. Various partition functions arise in the representation theory of permutation groups (see [2]). For example, if $p$ is prime, then let $b_p(n)$ denote the number of partitions of a non-negative integer $n$ where the summands are not multiples of $p$. If $n$ is a positive integer, then $b_p(n)$ denotes the number of irreducible representations of the symmetric group $S_n$ over the finite field with $p$ elements [2, Lemma 6.1.2].

For $b_k(n)$, the number of partitions of $n$ into parts none of which is a multiple of $k$, the generating function is given by the infinite product

$$\sum_{n=0}^{\infty} b_k(n)q^n = \prod_{n=1}^{\infty} \frac{1-q^{kn}}{1-q^n}. \tag{1}$$

There are other important examples of partition generating functions which contain similar infinite products. In particular we shall consider certain partition generating functions which contain infinite products of the form

$$\prod_{1 \le n \equiv g \pmod{\delta}} (1-q^n) \prod_{1 \le n \equiv -g \pmod{\delta}} (1-q^n)$$

where $0 \le g \le \delta$. For example the two Rogers-Ramanujan identities (see [1]),

$$\sum_{n=0}^{\infty} \frac{q^{n^2+an}}{(1-q)(1-q^2)\cdots(1-q^n)} = \prod_{n=0}^{\infty} \frac{1}{\left(1-q^{5n+a+1}\right)\left(1-q^{5n+4-a}\right)},$$

where $a = 0$ or $1$, involve such products.

For $r_{g,\delta}(n)$ the number of partitions of $n$ into parts that are congruent to $\pm g \pmod{\delta}$ where $0 < g < \lfloor \frac{\delta+1}{2} \rfloor$, the generating function for $r_{g,\delta}(n)$ is given by the infinite product

$$\sum_{n=0}^{\infty} r_{g,\delta}(n)q^n = \prod_{1 \le n \equiv g \pmod{\delta}} \frac{1}{(1-q^n)} \prod_{1 \le n \equiv -g \pmod{\delta}} \frac{1}{(1-q^n)}. \tag{2}$$

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