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GENERALIZED PUISEUX EXPANSIONS AND THEIR GALOIS GROUPS

SANJU VAIDYA

Section 1. Introduction

Let k be an algebraically closed field of characteristic p and let X be an indeterminate. Let $k((X))$ be the quotient field of the ring of formal power series (no convergence involved) in X over the field k. The field $k((X))$ is called the *field of meromorphic functions of X over k*. It is well known that in case $p = 0$, the Puiseux field $\cup_{n=1}^{\infty} k((X^{\frac{1}{n}}))$ of all Puiseux expansions is an algebraic closure of the field $k((X))$. But if $p \neq 0$, this is not the case. Chevalley [3] proved that polynomial $Z^p - Z - X^{-1}$ does not have a root in the Puiseux field.

Abhyankar [1] introduced the notion of generalized Puiseux expansion and proved the factorization of the said polynomial $Z^p - Z - X^{-1}$ into generalized Puiseux expansions. Using this, Huang, a doctoral student of Abhyankar, constructed a *generalized Puiseux field* and proved that it contains an algebraic closure of the meromorphic series field. The generalized Puiseux field consists of functions from the set Q of all rational numbers to the field k with some conditions on their support. In greater detail, a function f from the set Q to the field k is in the generalized Puiseux field iff its support $S(f)$ is a well ordered subset of the set Q and there exists an integer $m = m(f)$ such that for every $\alpha \in S(f)$ we have $\alpha m = \frac{n_\alpha}{p^{i_\alpha}}$ for some integers n_α and i_α. Huang [4] proved many fascinating results for generalized Puiseux elements whose supports are subsets of the set $\{\frac{-1}{p}, \frac{-1}{p^2}, \ldots, \frac{-1}{p^i}, \ldots\}$. For instance, he proved a criterion which says that such elements are algebraic over the field $k((X))$ iff they are *periodical* in case the field k is equal to algebraic closure of its prime field.

In this paper, we will investigate some functions of the generalized Puiseux field that are algebraic over the meromorphic series field; moreover, we will calculate their Galois groups. It turns out that Galois group of certain functions over the meromorphic series field is a semidirect product of a cyclic group and a direct sum of p cyclic groups. We also exhibit functions whose Galois groups are dihedral group, a certain type of Burnside group and a direct sum of p cyclic groups. Additionally, we will extend the criterion of Huang to a certain type of functions of the generalized Puiseux field in case the field k is not equal to algebraic closure of its prime field. We will also extend Huang's criterion to some generalized Puiseux elements whose supports are contained in the set $\{\frac{-l_i}{p^i}: i \in N\}$, where $(l_i)_{i \in N}$ is a sequence of positive integers satisfying certain constraints.

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