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TWISTOR SPINORS ON CONFORMALLY FLAT MANIFOLDS

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1. Introduction

Twistor spinors were introduced by R. Penrose as solutions of a conformally invariant field equations in general relativity. In this paper we consider Riemannian spin manifolds carrying twistor spinors. Outside their zero set they can be seen as conformal analogues of *parallel spinors*. As an example, a twistor spinor with a zero exists on the standard sphere. Moreover, A. Lichnerowicz proved in [Li, Thm. 7] that the sphere with its standard conformal structure is the only *compact* Riemannian spin manifold carrying twistor spinors with zeros.

To a spinor field ϕ one can canonically associate a vector field V_ϕ as the dual of the 1-form $X \mapsto \sqrt{-1}\langle\phi, X \cdot \phi\rangle$, where the dot refers to the Clifford multiplication and the bracket is the canonical hermitian inner product on the space of spinors. The associated vector field of a twistor spinor is *conformal*; i.e., its local flow consists of conformal transformations. There are twistor spinors for which the associated conformal field is trivial as well as twistor spinors with non-trivial conformal field, for example on the standard sphere.

In a previous paper the authors showed the following:

THEOREM 1.1 [KR1, THM. A]. *If the Riemannian spin manifold (M, g) carries a twistor spinor with zero and with non-trivial conformal field then the manifold is conformally flat.*

In this paper we obtain a converse statement, more precisely we describe the global types of conformally flat manifolds, which carry twistor spinors with zero and with non-trivial conformal field.

Let (M, g) be a *conformally flat* Riemannian manifold; i.e., every point p has an open neighbourhood U, such that (U, g) is conformally equivalent to an open subset of Euclidean space. A conformally flat manifold is called *developable* if there is a conformal map δ: $M^n \longrightarrow S^n$ into the standard sphere; δ is uniquely determined up to a conformal diffeomorphism of the sphere; i.e., up to a *Möbius transformation*. It follows that a developable conformally flat manifold carries twistor spinors with zero and with non-trivial assciated conformal fields. The universal covering $(\widetilde{M}, g)$ of a

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