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ON INTERMEDIATE RICCI CURVATURE AND FUNDAMENTAL GROUPS

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Synge's Theorem states that a closed Riemannian n-manifold with positive sectional curvature is orientable if n is odd and has fundamental group of order 1 or 2 if n is even. Products of real projective spaces show that Synge's Theorem is false for positive Ricci curvature. On the other hand, there is some evidence which suggests that large manifolds with positive Ricci curvature resemble large manifolds with positive sectional curvature ([Ander1], [Cold1,2], [CheCol1,2], [Perl1,2]). There is also some evidence to the contrary ([Ander2], [Otsu]).

It will be shown here that Synge's Theorem remains valid for any manifold M with positive Ricci curvature provided the first systole, $sys_1\ M$ (i.e., the length of the shortest closed noncontractible curve) is sufficiently large.

THEOREM 1. *Let M be a complete Riemannian n-manifold with $Ric\ M \geq n-1$ and $sys_1\ M > \pi\sqrt{\frac{n-2}{n-1}}$.*

(i) *If n is even and M is orientable, then M is simply connected.*
(ii) *If n is odd, then M is orientable.*

It is easy to see that a nonsimply connected, complete, Riemannian n-manifold with $Ric\ M \geq n-1$ has $sys_1\ M \leq \pi$ and that equality holds only if M is isometric to RP^n. Indeed if $sys_1\ M \geq r$, then the diameter of the universal cover $\tilde{M}$ is $\geq r$, so r must be $\leq \pi$ by the Bonnet-Myers Theorem. If $r = \pi$, then $\tilde{M}$ is isometric to S^n by [Cheng], and M is easily seen to be RP^n (cf. [Wil1]). By combining results in [Cold1,2], [CheCol2], and [FukYam] with an idea from [Wil1,2] it is also easy to see the following.

> *Given $n \in \mathbb{N}$ there is an $\varepsilon(n) > 0$ so that a complete, nonsimply connected, Riemannian n-manifold M with $Ric\ M \geq n-1$ and $sys_1\ M \geq \pi - \varepsilon$ is diffeomorphic to RP^n.*

(See the end of the paper for a sketch of the proof.)

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