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PERTURBATION OF PLANE CURVES AND SEQUENCES OF INTEGERS

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1. Perturbation of a curve

Definition 1.1. The Lebesgue measure on $\mathbb{R}^2$ is denoted by m.

Let Γ: $[0, \infty) \to \mathbb{R}^2$ be a continuous curve. For $s > 0$ and locally integrable f: $\mathbb{R}^2 \to \mathbb{C}$ we set

$$M_s f(x) = M_s(\Gamma, f)(x) = \frac{1}{s} \int_0^s f(x + \Gamma(t))dt.$$

(The measurability of $M_s f(x)$ is discussed in the appendix.)

Let $p \geq 1$. We say that Γ *differentiates* L^p_{loc} if and only if for $f \in L^p_{loc}(\mathbb{R}^2)$ we have

$$\lim_{s \to 0} M_s(\Gamma, f)(x) = f(x)$$

for m-a.e. $x \in \mathbb{R}^2$.

Let $1 \leq p < \infty$. We say that Γ is ∞-*sweeping out for* L^p if and only if there is $f \in L^p(\mathbb{R}^2)$ so that

$$\limsup_{s \to 0^+} M_s(\Gamma, f)(x) = \infty$$

for a.e. $x \in \mathbb{R}^2$.

We say that the continuous curve Δ: $[0, \infty) \to \mathbb{R}^2$ is a *perturbation* of Γ if and only if

$$\lim_{s \to 0} \frac{1}{s} \quad |\{t \mid 0 \leq t \leq s, \quad \Gamma(t) \neq \Delta(t)\}| = 0,$$

where $|A|$ means the one dimensional Lebesgue-measure of the set A.[1]

In the sequel C will denote a "generic" positive constant, which is independent of those quantities it should be independent of, but it can have different values even in the same set of inequalities.

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[1]We will use the same notation for the absolute value, but it will not cause any confusion.