ILLINOIS JOURNAL OF MATHEMATICS
Volume 42, Number 1, Spring 1998

CONTINUITY AND ANALYTICITY OF FAMILIES OF SELF-ADJOINT DIRAC OPERATORS ON A MANIFOLD WITH BOUNDARY

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Section 1

Given a continuous or analytic family D_t of self-adjoint elliptic operators on a manifold X, it is often useful to know whether the *spectrum* of D_t also varies continuously or analytically. If X is a closed manifold, the answer to this question is well known to be yes (assuming, in the analytic case, that the parameter space is an interval in the reals). The key point here is that because the domain of the operator is independent of the parameter t, one may apply standard theorems on deformations of self-adjoint operators to conclude that the spectrum varies in as nice a way as the operator. An excellent reference for these theorems is Kato's book, *Perturbation Theory For Linear Operators* [K].

If X is a compact manifold with boundary the situation is not as simple because one must choose boundary conditions in order for the operator to be self-adjoint; if these boundary conditions vary with the value of the parameter then the domain of the operator is changing. It is reasonable to expect a theorem that states essentially that given a continuous (resp. analytic) family of formally self-adjoint operators on a manifold X with boundary, and a continuous (resp. analytic) path of self-adjoint boundary conditions, one may conclude the the spectrum varies continuously (resp. analytically). In the context of Dirac operators and Atiyah-Patodi-Singer (APS) boundary conditions, this is precisely what our main theorem says.

THEOREM. *Let T be a topological space and $D_t = D_0 + A_t$ a family of formally self-adjoint Dirac operators on an odd-dimensional manifold with boundary in cylindrical form near the boundary. Suppose that the dimension of the kernel of the tangential operator $\hat{D}_t$ is independent of t. Choose a family $L(t)$ of Lagrangians in the kernel of $\hat{D}_t$. Then*:

1. *If the map $t \mapsto A_t$ is a continuous map into the space of smooth bundle endomorphisms and the family $L(t)$ is continuous then the self-adjoint family obtained by imposing APS boundary conditions on D_t is continuous in the graph topology; in particular the spectrum of D_t with $P_+(t) + L(t)$ boundary conditions depends continuously on t.*

Received March 5, 1997.
1991 Mathematics Subject Classification Primary 58G20; Secondary 58G25, 58G18.
Both authors partially supported by a grant from the National Science Foundation.