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ON CHEN'S BASIC EQUALITY

MARCOS DAJCZER AND LUIS A. FLORIT

Given an isometric immersion $f\colon M^n \to \mathbb{Q}_c^{n+p}$ of a riemannian manifold into a space of constant sectional curvature c, it was shown by B. Y. Chen [Ch1] that the inequality

$$\delta_M \leq \frac{n-2}{2}\left\{\frac{n^2}{(n-1)}\|H\|^2 + (n+1)c\right\} \tag{1}$$

holds pointwise. Here, H denotes the mean curvature vector of f and δ_M stands for the intrinsic invariant defined as

$$\delta_M(x) = s(x) - \inf\{K(\sigma)\colon \sigma \subset T_xM\},$$

where K and s denote, respectively, sectional and not normalized scalar curvature of M^n.

It is then natural to try to understand all submanifolds for which equality in (1) holds everywhere. In euclidean space, Chen showed that the *trivial* examples satisfying his *basic equality* are either affine subspaces or rotation hypersurfaces obtained by rotating a straight line, that is, cones and cylinders. Nontrivial examples for $n \geq 4$ divide in two classes, namely, *any* minimal submanifolds of rank two, which we completely describe in [DF], and a certain class of nonminimal submanifolds foliated by totally umbilic spheres of codimension two.

In this paper, we show that connected elements in Chen's second nontrivial class have the simplest possible geometric structure among submanifolds foliated by totally umbilic spheres, namely, they are rotation submanifolds over surfaces. This means that M^n is isometric to an open subset of a warped product $L^2 \times_\varphi \mathbb{S}_1^{n-2}$, $\varphi \in C^\infty(L)$ positive, and

$$f(x, y) = (h(x), \varphi(x)\, y) \tag{2}$$

being $h\colon L^2 \to \mathbb{R}^{p+1}$ a surface and $\mathbb{S}_1$ denotes a unit sphere. The surface $k := (h, \varphi)\colon L^2 \to \mathbb{R}^{p+2}$ is the profile of f.

The paper is organized as follows. First, we discuss the general problem whether a submanifold foliated by totally umbilic spheres of codimension two is rotational, and present necessary and sufficient conditions for this to occur. Then, we see that submanifolds satisfying the basic equality are either minimal or fall under those conditions. Finally, we present the restrictions for f in (2) in order to satisfy the basic equality. In particular, we show that rotational hypersurfaces over surfaces satisfying