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# MODEL THEORY OF PROFINITE GROUPS HAVING THE IWASAWA PROPERTY[1]

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## Introduction

The notion of complete systems of finite groups first appeared in a paper by Cherlin, Van den Dries and Macintyre [CDM], where it was used to give invariants for the theory of regularly closed fields (see also the work of Eršov [E]).

To a profinite group $G$ they associate the complete system $S(G)$, which encodes the inverse system of all finite (continuous) quotients of $G$ together with the projection maps. In an appropriate language, the systems $S(G)$ are $\omega$-sorted structures and form an elementary class. The connection with field theory is obtained as follows: for $K$ a field and $G(K)$ the absolute Galois group of $K$ (i.e., the Galois group of $K$ in its separable closure), the theory of the system $S(G(K))$ is interpretable in $\mathrm{Th}(K)$, and is in some sense the strongest such theory.

Besides field theory, complete systems can also be used to study profinite groups. Their main advantage is that one replaces the study of a group together with its topology, by the study of a fairly simple algebraic system. An other advantage is that by dualizing, one works with embeddings of complete systems instead of continuous epimorphisms of profinite groups.

The profinite groups we are interested in are the profinite groups having the Iwasawa property (IP). This property was first discovered by Iwasawa [I], who used it to characterize countably generated free profinite groups. This property was then considered by Cherlin, Van den Dries and Macintyre [CDM], and by Haran and Lubotzky [HL], among others.

The main result concerning the Iwasawa property given in [CDM], is that $\mathrm{Th}(S(G))$ is $\aleph_0$-categorical when $G$ has the Iwasawa property. It turns out that the types are easy to describe, and that $\mathrm{Th}(S(G))$ is $\omega$-stable. This allows one to use all the existing stability theoretic machinery in the study of these groups.

Besides characterizations of some model-theoretic properties, the main algebraic results obtained in this paper are:

THEOREM 2.6. *Let $H$ be a profinite group. Then $H$ has a universal* IP*-cover $G$, which is unique up to isomorphism over $H$.*

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[1]Most of this work is part of the author's Ph. D. thesis, Yale University, 1984.