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DISCRETIZATION OF LINEAR OPERATORS ON $L^p(\mathbb{R}^N)$

MARIA J. CARRO

1. Introduction

We say that the boundedness of an operator T: $L^p(\mathbb{R}^N) \to L^p(\mathbb{R}^N)$ can be discretized if we can characterize it by the boundedness of a collection of operators T_n on ℓ^p. Throughout this paper, we shall work under the restriction $1 < p < \infty$. There are many results of this type in the literature:

(a) Using simple estimates and the density of the simple functions on $L^p(\mathbb{R}^N)$, one can obtain that the boundedness of a linear operator on $L^p(\mathbb{R})$ is equivalent to the boundedness on ℓ^p of the operators associated to the matrices

$$\left(\left\langle 2^{kN/p} T\left(\chi_{(0,1)}(2^k \cdot -n)\right), 2^{kN/p'} \chi_{(0,1)}(2^k \cdot -m)\right\rangle\right)_{n,m},$$

uniformly in $k \in \mathbb{Z}$.

(b) Using Shannon's sampling theorem (see §3) one can show that the boundedness of a linear operator on $L^p(\mathbb{R}^N)$ is equivalent to the boundedness on $\ell^p(\mathbb{Z}^N)$ uniformly in k of the operator associated to the matrix

$$\left(\langle T(2^{kN/p}\text{sinc }(2^k \cdot -n)), 2^{kN/p'}\text{sinc }(2^k \cdot -m)\rangle\right)_{n,m},$$

with $\text{sinc } x = \prod_j \frac{\sin \pi x_j}{\pi x_j}$.

(c) In the context of Wavelet theory (see [M1], [M2]), the boundedness of a linear operator on $L^2(\mathbb{R})$ is equivalent to the boundedness on $\ell^2(\mathbb{Z}^2)$ of the operator

$$(a_{n,k})_{n,k} \to \left(\sum_{n,k} \left\langle 2^{kN/p} T(\varphi(2^k \cdot -n)), 2^{k'N/p'} \phi(2^{k'} \cdot -m)\right\rangle a_{n,k}\right)_{m,k'},$$

where φ and ϕ are wavelets. In [M2], they use this result to give a proof of the T1 theorem for singular operators.

(d) A result of de Leeuw and Jodeit (see [D] and [J]) shows that if $\text{supp}\, m \subset (-1/2, 1/2)^N$ and $\hat{K} = m$, then m is a multiplier in $L^p(\mathbb{R}^N)$ if and only if the sequence $(K(n))_n$ gives a convolution kernel on ℓ^p.

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