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INTERPOLATION OF HERZ-TYPE HARDY SPACES

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1. Introduction

To study convolution algebras, Beurling in [4] first introduced some spaces of functions which are now called the Beurling algebras. Later Herz in [19] generalized these function spaces to further study the properties of functions. These generalized spaces of functions are just the prototype of Herz spaces. Since then, the theory of Herz spaces has been significantly developed and these spaces have turned out to be very useful in analysis. An interesting account with many applications for the generalized Herz spaces in some particular cases is given in [2]. In particular, in [18] the authors of this paper characterized the intermediate spaces obtained by the complex method of interpolation for the families of Herz spaces and gave many interesting applications.

On the other hand, in recent years, a theory of Herz-type Hardy spaces has been developed (see [7], [14], [15], [22]–[25]). These new Hardy spaces are a sort of local version of the ordinary Hardy spaces and are good substitutes for the latter when considering, for example, the boundedness of non-translation invariant singular integral operators (see [26]). In this paper, we are going to characterize the intermediate spaces obtained by applying the complex method of interpolation to the families of Herz-type Hardy spaces and to the mixed couples of Herz spaces and Herz-type Hardy spaces.

Let us first introduce some notation. Let $B_k = \{x \in \mathbb{R}^n\colon |x| \leq 2^k\}$ and $C_k = B_k \setminus B_{k-1}$ for $k \in \mathbb{Z}$. Let $\chi_k = \chi_{C_k}$ for $k \in \mathbb{Z}$, $\widetilde{\chi}_k = \chi_k$ if $k \in \mathbb{N}$ and $\widetilde{\chi}_0 = \chi_{B_0}$, where χ_{C_k} is the characteristic function of the set C_k.

Definition 1.1 [19]. Let $\alpha \in \mathbb{R}$, $0 < p \leq \infty$ and $0 < q \leq \infty$.

(a) The homogeneous Herz space $K_q^{\alpha,p}(\mathbb{R}^n)$ is defined by

$$K_q^{\alpha,p}(\mathbb{R}^n) = \{f \in L^q_{\mathrm{loc}}(\mathbb{R}^n \setminus \{0\})\colon \|f\|_{K_q^{\alpha,p}(\mathbb{R}^n)} < \infty\},$$

where

$$\|f\|_{K_q^{\alpha,p}(\mathbb{R}^n)} = \left\{\sum_{k=-\infty}^{\infty} 2^{k\alpha p}\|f\chi_k\|^p_{L^q(\mathbb{R}^n)}\right\}^{1/p}.$$

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