SEMISIMPLICIAL SPECTRA

BY

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1. Introduction

The notion of a spectrum, introduced by Lima [12], has proved useful in homotopy theory. Spanier [17] used it for the study of stable homotopy theory, while E. H. Brown, Jr. [1] and G. W. Whitehead [18] have shown that there is a very close relationship between spectra and (co-) homology theories which satisfy all Eilenberg-Steenrod axioms but the dimension axiom.

Our present purpose is to define spectra in the semisimplicial context. Although the notion of a (topological) spectrum is a rather complex one (a sequence of spaces and maps), it turns out that a semisimplicial spectrum consists of only one object which very much looks like a semisimplicial complex with base point. The main differences are (i) that simplices are also allowed to have negative dimensions, and (ii) that every simplex y has an infinite number of faces $d_0 y$, $d_1 y$, $\cdots$ (but only a finite number of them are not "at the base point") and an infinite number of degeneracies $s_0 y$, $s_1 y$, $\cdots$.

Some applications will be given in [10] and [11].

There are two chapters. Chapter I deals with semisimplicial spectra and their relation to topological spectra. We also consider group spectra and show that the category of abelian group spectra is isomorphic with the category of abelian chain complexes.

In Chapter II a homotopy relation is introduced in the category of semisimplicial spectra. As for semisimplicial complexes this relation is, in general, not an equivalence relation. However on a suitable subcategory (that of spectra which "satisfy the extension condition") the homotopy relation is an equivalence relation. Consequently one has in this category the notions of homotopy equivalence and homotopy type. We end with considering minimal spectra and homotopy groups.

1.1 *Notation and terminology.* We shall freely use the results of [7] and [8] with the following changes in notation and terminology:

(i) the face and degeneracy operators will be denoted by d_i and s_i and will be written on the *left*;

(ii) c.s.s. complexes will be called *set complexes*, or for short, *complexes*; c.s.s. groups will be called *group complexes*.

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