MARKOV PROCESSES ON AN ENTRANCE BOUNDARY[1]

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This paper presents, essentially, an alternative approach to the second part of [7] in terms of "boundary theory." In [7, Section 2] the motivating idea was to regularize a temporally homogeneous Markov process $X(t)$, having as state space a set X, by using martingale theorems, independently of a topology on X or on the state space Y of the resulting process $Y(t)$. It is shown here that there is an alternative state space Y_ρ for $Y(t)$, and a compact metric topology on Y_ρ, such that $Y(t)$ (after a slight adjustment of the definition in [7] on certain sets of probability 0) has right continuous path functions. The necessary metric ρ is defined in a manner closely resembling that used by R. S. Martin [8] in defining a general boundary for the positive harmonic functions on Euclidean domains. In this way it is an extension of the boundaries discussed especially by Doob [2], Hunt [6], and Watanabe [10]. In terms of the metric ρ, the space Y_ρ is the completion of X, and the new process, denoted by $Y_\rho(t)$, is defined from $X(t)$ as the value of the process $X(t)$ on the "entrance boundary" corresponding to t, for each t. Equivalently, $Y_\rho(t)$ is simply given almost surely for all t by $Y_\rho(t) = \lim_{\tau \downarrow t, \tau \text{rational}} X(\tau)$, the limit being taken in the metric ρ.

In the second section we show the connection between this regularization of $X(t)$ and the general methods of Ray [9] for regularization of transition functions and processes corresponding to a given resolvent R_λ, $\lambda > 0$, operating on continuous functions. It may be pointed out that, aside from reasons of completeness, this connection is significant because, whenever $Y_\rho(t)$ is an instance of Ray's method, the theorems of [9] provide a transition function for $Y_\rho(t)$ together with a number of its properties which are not established otherwise.

Section 1

It is assumed in [7, Section 2] that a Markov process $X(t)$ relative to completed σ-fields $\mathfrak{F}(t)$ on a probability space (Ω, F, P) is given, together with a homogeneous transition function $p(t, x, E)$, $x \in X$, $E \in \mathfrak{B}$, for $X(t)$, and such that (a) $\mathfrak{B}$ is generated by countably many sets, (b) wide-sense conditional distributions over $\mathfrak{B}$ exist, (c) $p(t, x, E)$ is measurable in (t, x) over $\mathfrak{R} \times \mathfrak{B}$ where $\mathfrak{R}$ is the field of real Borel sets, and (d) $p(\cdot, x_1, \cdot) = p(\cdot, x_2, \cdot)$ implies that $x_1 = x_2$. These hypotheses are assumed again here. Under these hypotheses there is constructed in [7] a corresponding process $Y(t)$ which has for its range (at each t) families of joint distributions $F(t', E; t, w)$, $t' > 0$, $E \in \mathfrak{B}$, the "conditional futures", such that for

Received December 14, 1961.
[1] This research was supported in part by an Air Force Contract.