JACOBIANS AND SYMMETRIC PRODUCTS

BY

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The n-fold symmetric product $C(n)$ of a curve C is usually the starting point for the construction of the Jacobian variety J of C. Adopting this point of view, Mattuck [6] has determined the Chern classes of $C(n)$ regarded, for $n > 2g - 2$, as a projective fibre bundle over J. This determination led him to a set of intersection relations among the subvarieties of J which he conjectured should arise from an exact sequence of vector bundles on J.

In order to prove this conjecture, it is convenient to adopt a point of view exactly opposite to that mentioned above. Namely, we assume the existence, for a complete nonsingular algebraic curve C, of a Picard variety J satisfying the general properties used by Lang [5] to define the Picard variety. We then define certain sheaves on J which we call (following Mattuck [7]) Picard sheaves, and prove certain properties of exactness and duality which they satisfy. This is enough to obtain the intersection relations of Mattuck to which we alluded above. The advantage of this point of view is that it is not necessary to go over again any of the steps in the construction of the Jacobian, and hence that some (but not all) of the theory will extend to any Picard variety.

We shall make considerable use of certain constructions contained in the *Éléments* of Grothendieck [4], in particular, the construction which associates with any coherent sheaf $\mathcal{E}$ a fibred variety $\mathbf{P}(\mathcal{E})$. It is this construction which enables us, finally, to reconstruct the symmetric products $C(n)$ from the Picard sheaves.

We give references to [4] whenever the relevant chapter is already available, but do not intend to imply that they cannot be found elsewhere. Nor do we, in giving yet another aspect of the link between Jacobians and symmetric products, wish to slight the rich literature which already exists on the subject, and to which references will be found in [5], [6], [7], [8]. We adopt the following notations involving coherent sheaves. If $f: Y \to X$ is a regular map and $\mathfrak{F}$ is a coherent sheaf on X, we write $\mathfrak{F}^* = \mathrm{Hom}(\mathfrak{F}, \mathcal{O}_X)$ and $f^*\mathfrak{F} = \mathfrak{F} \otimes \mathcal{O}_Y$. If $\mathcal{G}$ is a coherent sheaf on Y, we write $f_r(\mathcal{G})$ for the sheaf with presheaf $f_r(\mathcal{G})(U) = H^r(f^{-1}(U), \mathcal{G})$. If V is a vector space, we write $\mathbf{P}(V)$ for the projective space whose points correspond to the hyperplanes of V. This construction is extended in [4, II, 4.1] to an arbitrary coherent sheaf.

1. Algebraic curves

Let C be a complete nonsingular curve of genus g defined over an algebraically closed field k, and $c \in C$ a fixed base point. For each integer s, define

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