THE MODULAR REPRESENTATION ALGEBRA OF A FINITE GROUP

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1. Representation algebras

1.1. *Notation and terminology.*

G is a finite group, with unit element e.

k is a field of characteristic p.

By a *G-module* M is meant a (k, G)-module. Elements of G act as right operators on M, and $me = m$ $(m \in M)$. The k-dimension $\dim M$ of M is assumed finite. For example,

$\Gamma = \Gamma(k, G)$ is the *regular G-module*, i.e., the group algebra of G over k, regarded as G-module, and

k_G is the *unit G-module*, i.e., the field k, made into a "trivial" G-module, i.e., $\kappa x = \kappa$ $(\kappa \in k, x \in G)$. For any G-module M,

$\{M\}$ is the class of all G-modules isomorphic to M.

V_i (i runs over a suitable index set I) is a set of representatives of the classes $\{V_i\}$ of indecomposable G-modules. The number of these indecomposable classes is finite if and only if either $p = 0$, or p is a finite prime such that the Sylow p-subgroups of G are cyclic (D. G. Higman [5]).

F_j $(j = 1, \cdots, n)$ is a set of representatives of the classes $\{F_j\}$ of irreducible G-modules. The number n of these is always finite. If k is algebraically closed, n is equal to the number of p-regular classes of G (R. Brauer, see [1], [2]).

If M', M'' are G-modules, $M' + M''$ denotes their *direct* sum. If M is a G-module, and s a nonnegative integer, sM denotes the direct sum of s isomorphic copies of M.

1.2. Let $\mathfrak{c}$ be an arbitrary commutative ring with identity element. Then the *representation algebra* $A_{\mathfrak{c}}(k, G)$ of the pair (k, G), with coefficients in $\mathfrak{c}$, is defined as follows. It is the $\mathfrak{c}$-module generated by the set of all isomorphism classes $\{M\}$ of G-modules, subject to relations $\{M\} = \{M'\} + \{M''\}$ for all M, M', M'' such that $M \cong M' + M''$, and equipped with the bilinear multiplication given by $\{M\}\{M'\} = \{M \otimes M'\}$. Here $M \otimes M' = M \otimes_k M'$ is made G-module by $(m \otimes m')x = mx \otimes m'x$ $(m \in M, m' \in M', x \in G)$. By the Krull-Schmidt theorem for G-modules, $A_{\mathfrak{c}}(k, G)$ is free as $\mathfrak{c}$-module, and the $\{V_i\}$ $(i \in I)$ form a $\mathfrak{c}$-basis. $A_{\mathfrak{c}}(k, G)$ is a commutative, associative algebra over $\mathfrak{c}$, and has identity element $1 = \{k_G\}$.

The *Grothendieck algebra* $A_{\mathfrak{c}}^*(k, G)$ is the quotient of $A_{\mathfrak{c}}(k, G)$ by the ideal J

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