MATHEMATICAL CHARACTERIZATION OF THE PHYSICAL VACUUM FOR A LINEAR BOSE-EINSTEIN FIELD[1]

(Foundations of the dynamics of infinite systems III)

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The mathematical treatment of the physical vacuum is one of the most challenging and basic problems of quantum field theory. In the case of "interacting" fields this problem is in substantial part one of the formulation of the underlying theory, but in the case of "free" fields—which are theoretically enlightening as well as physically relevant by virtue of the mathematical identity of their structure with that of an interacting field in the "interaction" representation at a particular time, this structure being empirically manifested, according to the usual postulates, at the times $\pm\infty$— the necessary basic formulations are now at hand. To take the presently most conservative and general position, such a field is mathematically a somewhat structured C^*-algebra of "observables" (representable, but not at all in any unique way, as a uniformly closed self-adjoint algebra of bounded linear operators on a Hilbert space), and the physical states are certain linear forms on this algebra, having the conventional interpretation of the expectation value form corresponding to the given state. This assumption is entirely independent of any assumptions as to the nature of space-time, or group-invariance of the theory.

The present paper is concerned with the problem of characterizing, in terms which are both mathematically rigorous and physically meaningful, the physical vacuum for such general types of field. The conventional formulation of the vacuum in theoretical physics as "the state of lowest energy" can be immediately transcribed mathematically, but in such a nonunique manner as to be ineffective except for certain formal purposes. The well-known divergences of quantum field theory signify essentially that the energy in conventional theories of interacting fields is mathematically highly ambiguous. It is less familiar, but equally troublesome, that the Hilbert space on which the energy is supposed to act as a self-adjoint operator, has no explicit formulation in the conventional theories. Both of these difficulties are connected with the existence of many inequivalent representations for the canonical variables of a quantum field [1], although in the relatively transparent case of a free field there are no nontrivial divergences. Nevertheless, the conventional description of the free field involves either mathematical ambiguity or technical requirements lacking in physical interpretation. The more conservative approach described above avoids the problem posed by the ambiguity of the

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