UNIQUENESS OF INVARIANT WEDDERBURN FACTORS

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1. Introduction

In this note, an affirmative answer is given to a conjecture which the author made in the last section of [4]. Let A denote a finite-dimensional associative algebra over a field Φ. Let N denote the radical of A, and let A/N be a separable algebra. Then it is well known (the Wedderburn principal theorem) that A possesses a separable subalgebra S such that $A = S + N$, $S \cap N = \{0\}$, and $S \cong A/N$. In [4], we showed that if G is a finite group, each of whose elements is either an automorphism or an antiautomorphism of A, and whose order is not divisible by the characteristic of Φ, then the subalgebra S described above may be chosen to be invariant under the operators in G, i.e., a G-subalgebra. In general, the Malcev theorem [3] states that if S and T are two such separable subalgebras (called Wedderburn factors), then there exists an (inner) automorphism of A which carries S onto T. In [4], we conjectured that if S and T are two G-invariant Wedderburn factors, then there exists an automorphism of A, carrying S onto T, which commutes with each operator in G, i.e., a G-automorphism. In Section 4 of [4], this was proved for the special case of characteristic Φ equals zero, and G consisting of an involution of A and the identity mapping of A. Here we establish the conjecture for an arbitrary finite group G for the case of characteristic Φ equals zero.

2. Preliminaries

We assume familiarity with the notions of a nilpotent derivation, and the adjoint mapping of A into its Lie algebra of derivations. In particular, if $z \in N$, then $\exp z$ is regular (in A_1, the algebra obtained from A by adjunction of an identity, if necessary), and $\exp(\operatorname{Ad} z)$ is the inner automorphism determined by conjugation by $\exp z$.

We first note that if G contains an element which is both an automorphism and an antiautomorphism, then A is commutative. In this case, since the automorphism given by the Malcev theorem is inner, there is a unique Wedderburn factor, so that the desired result is trivial. Hence we now assume that A is not commutative, and that each element of G is either an automorphism of A or an antiautomorphism of A, but not both.

If $\tau \in G$, we extend τ to A_1 by setting $\tau(\alpha 1) = \alpha 1$ for $\alpha \in \Phi$. If $z \in A_1$, we call z G-symmetric if $\tau z = z$ for $\tau \in G$, τ an automorphism of A, and $\tau z = -z$ for $\tau \in G$, τ an antiautomorphism of A. It is easy to verify that

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