EXISTENCE OF NORMAL COMPLEMENTS AND EXTENSION OF CHARACTERS IN FINITE GROUPS

Dedicated to Reinhold Baer on the occasion of his sixtieth birthday

BY

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The main purpose of this paper is to tie together the following problems and to find conditions under which they may be solved.

PROBLEM A. Given a finite group G and a Hall subgroup H, when is there a normal complement to H in G?

PROBLEM B. Given a finite group G and a Hall subgroup H, when is it possible to extend each of the irreducible characters of H to one of G?

Of course, a positive solution of Problem A for the groups G and H leads to a positive solution of Problem B.

In both problems, we may drop the restriction on H, but the example of an abelian group G shows that the extended problems are not equivalent.

Our main results are the following:

THEOREM 1. *Let G be a π-separated group. Then the following conditions are equivalent:*

(a) *G contains a normal π'-Hall subgroup.*

(b) *Each π-Hall subgroup of G is c-closed.*

(c) *At least one π-Hall subgroup of G is c-closed.*

THEOREM 2. *If H is a soluble Hall subgroup of G, then the following conditions are equivalent:*

(a) *G contains a normal complement to H.*

(b) *Each irreducible character of H may be extended to G.*

THEOREM 3. *Let H be a Hall subgroup of G such that at least one of the following conditions holds:*

(1) *H has a Sylow tower.*

(2) *The terminal member of the lower central series of H is nilpotent.*

Then, the following conditions are equivalent:

(a) *G contains a normal complement to H.*

(b) *H is c-closed in G.*

THEOREM 4. *If H and K are Hall subgroups of G of complementary orders, then the following conditions are equivalent:*

(a) *G is the direct product of H and K.*

(b) *Each irreducible character of H and of K may be extended to G.*

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