THE INITIAL VALUE PROBLEM FOR MAXWELL'S EQUATIONS FOR TWO MEDIA SEPARATED BY A PLANE[1]

Dedicated to Hans Rademacher on the occasion of his seventieth birthday

BY
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Let the column vectors

$$E = \begin{pmatrix} E_1 \\ E_2 \\ E_3 \end{pmatrix}, \qquad H = \begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix}$$

describe an electromagnetic field. Denoting the space coordinates by x_1, x_2, x_3 and the time by t we put

$$\xi_i = \partial/\partial x_i, \qquad \tau = \partial/\partial t.$$

The "curl" operator is then represented by the matrix

$$C(\xi) = \begin{pmatrix} 0 & -\xi_3 & \xi_2 \\ \xi_3 & 0 & -\xi_1 \\ -\xi_2 & \xi_1 & 0 \end{pmatrix},$$

while the "divergence" operator corresponds to the row vector

$$\xi = (\xi_1, \xi_2, \xi_3).$$

Maxwell's equations for a homogeneous, isotropic, nonconducting medium in the absence of charges then take the form

$$\varepsilon\tau E = C(\xi)H, \qquad \mu\tau H = -C(\xi)E, \qquad \xi E = \xi H = 0.$$

We consider now the case of two media separated by the plane $x_1 = 0$. The field in the medium $x_1 < 0$, where the electric capacities shall have values ε, μ, we denote by E, H. We require that

$$\text{(1a)} \quad \varepsilon\tau E = C(\xi)H, \qquad \mu\tau H = -C(\xi)E, \qquad \xi E = \xi H = 0 \quad \text{for } x_1 \leqq 0.$$

The field in the other medium, where the capacities shall have values ε', μ', we denote by E', H'. For our purposes it is convenient to use in the second field a new name x_1' for the first space coordinate x_1. Putting

$$\xi_1' = \partial/\partial x_1', \qquad \xi' = (\xi_1', \xi_2, \xi_3)$$

Received April 28, 1961.

[1] This paper represents results obtained at the Institute of Mathematical Sciences, New York University, sponsored by the Office of Naval Research, United States Navy.