# MONOTONE BEHAVIOR OF COHOMOLOGY GROUPS UNDER PROPER MAPPINGS[1]

BY
Ernst Snapper

## Introduction

The algebraic varieties which occur in this paper are defined over an algebraically closed groundfield of arbitrary characteristic. We use the terms algebraic variety, quasi-projective and projective variety, regular and proper mappings, coherent sheaves, etc., as defined in [1] and [2].

If $f:X \to Y$ is a continuous mapping from a topological space $X$ into a topological space $Y$, and $F$ a sheaf of abelian groups over $X$, the sheaves $R^q f(F)$ over $Y$ are well defined for all $q \geqq 0$; see [3], Section 3.7. (We write $f(F)$ instead of $R^0 f(F)$ for the direct image of $F$.) A spectral sequence is associated with $f$ and $F$ whose initial term is $E_2^{p,q}(F) = H^p(Y, R^q f(F))$ and whose final term is $E^n(F) = H^n(X, F)$; see [3], Theorem 3.7.3. Consequently, there exists a natural homomorphism $\alpha^n: H^n(Y, f(F)) \to H^n(X, F)$ for all $n \geqq 0$, which is a monomorphism for $n = 1$. The main theorem of this paper states:

Theorem 1. *Let $f:X \to Y$ be a proper mapping from an irreducible, quasi-projective variety $X$ onto an algebraic variety $Y$ of the same dimension $r$. Then, if $F$ is a coherent sheaf over $X$, the natural homomorphism*

$$\alpha^r: H^r(Y, f(F)) \to H^r(X, F)$$

*is an epimorphism.*

We observe that, since is $X$ is irreducible and $f$ is onto, $Y$ is irreducible. We do not know whether the theorem remains correct if we assume only that $X$ is an irreducible, algebraic variety which is not necessarily quasi-projective.

Many conclusions can be drawn from Theorem 1. For example, if we assume that the $Y$ in that theorem is normal and the $f$ is birational, $f(O_X) = O_Y$ ; $O_X$ and $O_Y$ denote the sheaves of local rings of, respectively, $X$ and $Y$. Theorem 1 then states that $\alpha^r: H^r(Y, O_Y) \to H^r(X, O_X)$ is an epimorphism which, in the special case that $X$ and $Y$ are projective, was proved by much more complicated methods on page 94 of [4]. If furthermore $r = 2$ and $X$ is projective (in which case $Y$ is necessarily complete, and all the cohomology groups under investigation are finite-dimensional vectorspaces over $k$), the epimorphism $\alpha^2$, together with the monomorphism

$$\alpha^1: H^1(Y, O_Y) \to H^1(X, O_X),$$

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