GEOMETRIC ABA-GROUPS

BY

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1. Introduction

A group of collineations of an incidence system will be called *acutely transitive* if it is transitive on the configurations consisting of an incident point and line. By an (*acutely transitive*) *representation* of a group G on an incidence system Σ will be meant a homomorphism of G onto an (acutely transitive) group of collineations of Σ.

A finite incidence system will be called a *2-design* if each point lies on the same number $h \geqq 2$ of lines, each line contains the same number $k \geqq 2$ of points, and each pair of points lies on exactly one line (of course, 2-designs are special balanced incomplete block designs). In this paper we characterize the finite groups admitting acutely transitive representations on 2-designs as the groups G containing subgroups A and B such that

(1) $G = ABA$,
(2) $AB \cap BA = A + B$, and
(3) $A \not\subseteq B$ and $B \not\subseteq A$.

Such a group we call a *geometric ABA-group*. Any doubly transitive group is a geometric ABA-group with $B:A \cap B = 2$. We observe that an acutely transitive group on a 2-design is necessarily primitive on the points, which means that in a geometric ABA-group, A is a maximal subgroup.

A finite group admits an acutely transitive representation θ on a finite projective plane π if and only if it is a geometric ABA-group satisfying

(3′) $A:A \cap B = B:A \cap B \geqq 3$,

in which case we call it a *projective ABA-group*. By using the Ostrom-Wagner theorem [7; Theorem 5] it is easy to see that then the additional condition

(4) $G = A + AxA$

is necessary and sufficient for π to be Desarguesian and $\theta(G)$ to contain the little projective group. As an application we show that a simple group satisfying Steinberg's axioms [10] with the symmetric group of degree 3 as Weyl group is necessarily a little projective group. We show that if a projective ABA-group has $A:A \cap B = n + 1$, where n is either an odd nonsquare, or $n = n_0^2$ with $n_0 \equiv -1 \pmod 4$, then the plane is Desarguesian, and $\theta(G)$ contains the little projective group.

Received August 29, 1960.

[1] Part of the work on this paper was done while the first author was supported by an Air Force contract.