SOME GENERALIZATIONS OF FINITE PROJECTIVE DIMENSION

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1. Introduction

In this paper we shall deal with various generalizations of the concept of finite projective dimension. Modules of finite projective dimension and rings of finite global dimension have been extensively studied by a number of authors; see, for instance, [1, 3, 4, 5, 6]. These studies have added significantly to our knowledge of rings with minimum condition. The chief drawback, however, to the concept of finite projective dimension is that not enough modules have it. It is for this reason that we initiated this investigation into various generalizations.

Throughout the paper, module will mean finitely generated module, and ring will mean ring with minimum condition on the kind of ideals that match the modules. Since we do not change rings, we will write the functors Hom and Ext without mention of the rings. Notation and terminology will follow that of [3].

In Section 2 we develop a theory on "how many" modules are necessary for a projective resolution. The results in that section rest heavily on the work of Eilenberg concerning minimal resolutions. A handy tool in the study of minimal resolutions is the use of $\operatorname{Ext}(A, C)$ as a module over the endomorphism ring of C where C is irreducible. In this case, $\operatorname{Ext}(A, C)$ becomes a vector space, and we can count modules in the projective resolution for A by computing the dimensions of certain of these vector spaces.

In Section 3 we prove two versions of Nunke's theorem [10] for the type of modules under consideration. There, the class of rings for which we prove the theorem is more general than the class of rings with minimum condition with finite global dimension. However, we only consider finitely generated modules. Actually the theorem can be proved from the work of Auslander [1] for semiprimary rings of finite global dimension without the assumption of finite generation of the modules involved. Our method of proof works only for finitely generated modules.

2. Projective types

In this section, we shall be concerned with "how many" projective modules appear in the minimal projective resolution of a module. Because of the restrictions that we have imposed on the rings and modules that we are considering, we have the following facts at our disposal:

Received July 18, 1960.

[1] The author gratefully acknowledges the support of the National Science Foundation.