RANDOM WALKS WITH ABSORBING BARRIERS AND TOEPLITZ FORMS

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1. Introduction

In a recent paper Spitzer and Stone [11] considered some asymptotic properties of the Toeplitz matrices

$$\| T(N)_{k,j} \| = \| c_{j-k} \| \qquad (k, j = 0, 1, \cdots, N)$$

where the c_k satisfied

$$c_k = c_{-k} \geqq 0, \qquad k = 0, 1, \cdots, \tag{1.1}$$

$$\textstyle\sum_{k=-\infty}^{+\infty} c_k = 1, \tag{1.2}$$

$$\text{g.c.d. } [k \mid k > 0,\ c_k > 0] = 1, \tag{1.3}$$

$$0 < \textstyle\sum_{k=-\infty}^{+\infty} k^2 c_k < \infty. \tag{1.4}$$

By (1.1) and (1.2)[1]

$$c_k = P\{X = k\} \tag{1.5}$$

defines a probability distribution of a random variable X, and consequently most of the results in [11] have an easy probability interpretation. Putting

$$S_n = X_0 + \textstyle\sum_{k=1}^{n} X_k ,$$

where $X_1, X_2, \cdots$ is a sequence of independent random variables, each distributed as X in (1.5), and X_0 an unspecified integer, it was shown in [11] that

(1.6) $H(N)_{k,j} = [I - T(N)]_{k,j}^{-1} =$ Expected number of visits to j of the S_n process with $S_0 = X_0 = k$ before leaving the interval $[0, N]$.[2]

One also has ([11])

$$H(N)_{k,j} = \textstyle\sum_{r=\max(k,j)}^{N} p_{r,k}\, p_{r,j} ,$$

where the $p_{r,k}$ are the coefficients of the orthogonal polynomials corresponding to the weight function

$$1 - \phi(t) = 1 - \textstyle\sum_{k=-\infty}^{+\infty} c_k \exp(ikt).$$

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[1] $P\{A\}$ = the probability of the event A,
$P\{A|B\}$ = the conditional probability of A, given B,
EX = expectation of the random variable X,
$E\{X|B\}$ = the conditional expectation of X, given B.

[2] $[a]$ = the largest integer $\leqq a$,
$[b, c]$ is the closed interval $b \leqq \xi \leqq c$.
This double use of square brackets is not likely to lead to confusion. (b, c) is the open interval $b < \xi < c$.