THE STRUCTURE OF CONTOURS OF A FRÉCHET SURFACE

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1. Introduction

The concept of a contour for a Fréchet surface was introduced by Cesari [1] who made use of contours to establish the Cesari-Cavalieri inequality, to develop some of the most fundamental properties of Fréchet surfaces, and to investigate variational problems for surface integrals. In a recent paper [3], Cesari and the author introduced several methods for smoothing contours by deleting certain inessential portions from them and proved the equivalence of several such methods. The method of contours is of value chiefly because it provides a means for constructing on the surface a conveniently disposed family of continuous curves. The counterimages of these curves, which are called contours, lie in the two-dimensional set over which the surface is defined, and it is the principal purpose of this paper to show that a representative mapping defining the surface can be found for which almost all of the contours have a simple structure. To this end we rely heavily on the methods and results of the previous paper [3] on smoothing methods for contours, and in Section 2 a brief exposition of smoothing methods will be given. In Section 3 we establish certain properties of smoothed contours and show that, in computing the length of the image of a smoothed contour, either an outer or inner border may be used. The principal result is established in Section 4 in which it is shown that for a nondegenerate surface of the type of the disk, a representation can be found for which almost all contours are arcs, points, or simple closed curves. This constitutes a considerable improvement over a previous result of the author [4] in which he showed that a countable dense set of contours had this property.

2. Notations and definitions

Let Q be a bounded, closed, simply connected, planar Jordan region, and let $T:Q \to E_N$ be a continuous mapping from Q into euclidean N-space. Then T defines a Fréchet surface S. We assume that S has finite Lebesgue area, and we denote by $[S]$ the set of points in E_N occupied by the surface. It may also be assumed that Q is the unit square in the (u, v) coordinate plane, $Q = \{p = (u, v) \mid 0 \leqq u, v \leqq 1\}$.

Let f be a real-valued continuous function defined on $[S]$ with upper and lower bounds t_1, t_2 respectively. For $t_1 \leqq t \leqq t_2$ we define $D^-(t)$, $D^+(t)$,

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