TWO-POINT BOUNDARY PROBLEMS INVOLVING A PARAMETER LINEARLY

BY

HYMAN J. ZIMMERBERG[1]

1. Introduction

The present paper is concerned with the extension of the concepts of adjointness, normality, symmetrizability, and definiteness of Bliss [1], [2] and Reid [4], [6] to linear differential systems, written in vector form

$$\begin{aligned} &y' - A(x)y = \lambda B(x)y, && a \leqq x \leqq b, \\ &(M_0 + \lambda M_1)y(a) + (N_0 + \lambda N_1)y(b) = 0. \end{aligned} \tag{1.1}$$

In addition, the hypotheses imposed on the coefficients of the boundary conditions are analyzed, and necessary and sufficient conditions for testing these assumptions for individual problems (1.1) are developed. For real-valued coefficients, Bobonis [3] has extended the class of definite problems introduced by Bliss [1] and [2] to problems (1.1) in which corresponding assumptions on the boundary conditions are postulated. As for problems with boundary conditions not involving the parameter, the extension of the definite classes of Reid [4] to problems (1.1) also yields further results for the definite problems of Bobonis.

Section 2 introduces the basic assumptions made on the coefficients of (1.1), and a simple necessary and sufficient test for the conditions imposed on the boundary conditions both in this paper and by Bobonis [3] to hold is given. Adjoint boundary problems and their basic interrelations with the original problem are also discussed. In Section 3 the equivalence of two boundary problems of the form (1.1) under a nonsingular transformation is discussed; in particular, the equivalence of a problem with its adjoint. A problem (1.1) will be termed abnormal if there exist nontrivial vectors $y(x)$ satisfying

$$\begin{aligned} &y' - A(x)y \equiv 0 \quad \text{and} \quad B(x)y \equiv 0 && \text{on } ab, \\ &M_0 y(a) + N_0 y(b) = 0 \quad \text{and} \quad M_1 y(a) + N_1 y(b) = 0, \end{aligned}$$

and otherwise normal. For abnormal problems (1.1) equivalent to their adjoint under a nonsingular skew-hermitian transformation, Theorem 6.1 of Reid [6] is extended in Section 4 to establish the existence of an equivalent normal problem also equivalent to its adjoint under the same transformation

Received October 19, 1959.

[1] This paper contains, in part, results obtained in the author's doctoral dissertation written under the guidance of Professor W. T. Reid. That portion of the results was presented to the American Mathematical Society on September 15, 1945; abstract no. 94, Bull. Amer. Math. Soc., vol. 51 (1945), p. 369. Subsequent results were presented to the American Mathematical Society on June 18, 1960.