GROUPS ON S^n WITH PRINCIPAL ORBITS OF DIMENSION $n-3$

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Introduction

Throughout this paper X is to be S^n with the usual differentiable structure, and G is to be a compact connected Lie group acting differentiably on X, with principal orbits of dimension $n-3$. We begin without assuming the existence of a stationary point and obtain some information about the orbit space X^*. However our main concern is with the case where a stationary point exists. In this case we obtain rather complete information about X^*. The main result is that if $n > 4$, then X^* is a 3-manifold with boundary S^2, and furthermore D is null, where D is the set of points on exceptional orbits of dimension $n-3$.

Let G_x be the isotropy group at x. The orbit $G(x)$ is a principal orbit if it is $(n-3)$-dimensional and if for y in a neighborhood of x, G_y and G_x have the same number of components. If $G(x)$ is $(n-3)$-dimensional and there is no such neighborhood of x, then $G(x)$ is called an exceptional three-dimensional orbit. The principal orbits form a dense open connected subset of S^n whose complement has dimension at most $n-2$. For all $(n-3)$-dimensional orbits, it is true that dim G_x is constant. For all principal orbits $G(x)$ the number of components of G_x is constant.

Let U be the union of all principal (sometimes regular) orbits, D the union of exceptional $(n-3)$-dimensional orbits, and B the union of all singular orbits. Then

$$X = U \cup D \cup B,$$

and the sets U, B, D are invariant and mutually exclusive. Let p be the natural map from X to the orbit space X^*.

The number of components of G_x is denoted by $m(x)$ and $m(x^*) = m(x)$, for any x such that $p(x) = x^*$. It will be convenient to use $\rho(y^*)$ where

$$\rho(y^*) = m(y^*)/m(x^*), \qquad y^* \in D^*, \quad x^* \in U^*.$$

It is known that U^* is orientable and that every orbit in $U \cup D$ is orientable [8].

In the case $n = 4$, G a circle, there is the following special example with a stationary point Let the circle act on one plane with period $1/p$, $p > 1$, and on another plane with period $1/q$, $q > 1$. Then G acts on the product of the planes by defining

$$g(p_1, p_2) = (g(p_1), g(p_2)),$$

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