QUOTIENT FIELDS OF RESIDUE CLASS RINGS OF FUNCTION RINGS

BY

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The ring of all continuous functions from a topological space X into the reals, $\mathbf{R}$, is denoted by C or $C(X)$. In [1, 14E], an example is presented of a residue class field of one ring of continuous functions that is isomorphic in a natural way with the quotient field of a residue class ring of another ring of functions. The first ring is $C(\mathbf{N})$, $\mathbf{N}$ denoting the discrete space of positive integers. The second is $C(\Sigma)$, where $\Sigma = \mathbf{N} \cup \{\sigma\}$ is the subspace of the Stone-Čech compactification of $\mathbf{N}$ obtained by adjoining a single point σ to $\mathbf{N}$. The set M of all functions in $C(\mathbf{N})$ that vanish on a set having σ in its closure is a maximal ideal in $C(\mathbf{N})$; the set Q of all functions in $C(\Sigma)$ that vanish on a neighborhood of σ is a nonmaximal prime ideal in $C(\Sigma)$. In the manner to be described in §2, the mapping that sends each function in $C(\Sigma)$ into its restriction to $\mathbf{N}$ induces an isomorphism of the integral domain $C(\Sigma)/Q$ onto a subring of $C(\mathbf{N})/M$, and $C(\mathbf{N})/M$ is the quotient field of that subring.

In the present paper, we investigate the possibility of obtaining, in a similar way, the quotient field of $C(Y)/Q$, where Q is a prime ideal in an arbitrary function ring $C(Y)$. We shall find that a necessary condition is that Q be a *z-ideal*, i.e., if $h \in C(Y)$, and if there exists $g \in Q$ such that $h(y) = 0$ wherever $g(y) = 0$, then $h \in Q$. A sufficient condition is that Q have an immediate successor in the family of all z-ideals in $C(Y)$. On the other hand, if Q is the intersection of a countable family of z-ideals different from itself, then the quotient field of $C(Y)/Q$ is not isomorphic with a residue class field of *any* function ring. The question is left open as to what may happen in case Q neither has an immediate successor nor is a countable intersection; whether such a prime z-ideal Q exists at all is also left unsettled.

1. Preliminaries

The terminology and notation of [1] will be used throughout the paper. In this section, we summarize the material from [1] that will be used. Most of the information about prime ideals can also be found in [2] and [3].

When dealing with algebraic properties of a ring $C(X)$, one loses no generality by supposing X to be completely regular. We adopt this standing assumption.

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