# ON THE GLOBAL STRUCTURE OF THE TRAJECTORIES OF A POSITIVE QUADRATIC DIFFERENTIAL

BY

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1. A knowledge of the global structure of the trajectories of a positive quadratic differential on a finite oriented Riemann surface is of fundamental importance in the proof of the General Coefficient Theorem [1, 2, 3, 4]. The first steps in this direction were taken by Teichmüller [9] who described the local structure of the trajectories at various critical points, as well as some of the basic types of domains comprising the global structure with, however, little indication of proof. His discussion was limited to some rather special hyperelliptic differentials (i.e., defined on the Riemann sphere). Schaeffer and Spencer [8] gave a fairly complete treatment of the local structure and analyzed the global structure for those differentials treated by Teichmüller, as well as for one other special hyperelliptic differential, by a method whose application is essentially restricted to these particular cases. In particular, it was not decided whether, in the case of a hyperelliptic differential, a trajectory could be everywhere dense in some domain. The first general results on global structure were given in a paper [7] by the author and Spencer, where it was shown that for a hyperelliptic differential the trajectory structure is made up of end, strip, circle, and ring domains [2; pp. 36, 37] together with a finite number of domains in which some of the trajectories having limiting end points at finite critical points of the differential are everywhere dense. It was shown by example that such domains can actually be present. Later the author remarked [1] that the same considerations apply on a general finite oriented Riemann surface, and a complete treatment of this characterization of the global structure is found in [2], where the results are summarized as the Basic Structure Theorem [2; Theorem 3.5]. This result is sufficient for proving the General Coefficient Theorem, but one somewhat unsatisfactory feature remains. This is the lack of knowledge of the structure within those domains where there are everywhere dense trajectories. The only information in this direction is contained in [7; §3]. Now the simplest prototype of everywhere dense structure occurs for everywhere regular quadratic differentials on a closed surface of genus one. If $Q(z)\,dz^2$ is one such differential, then all such are of the form $Ke^{i\theta}Q(z)\,dz^2$ with $0 \leqq \theta < 2\pi$, $K > 0$. For a countable set of values of $\theta$ each trajectory is a closed curve; for all other values each trajectory is everywhere dense on the full surface. In this paper we will show that those domains in which the everywhere dense structure occurs decompose into subdomains such that every trajectory in such a subdomain is everywhere dense.

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