THE DUAL OF A SECONDARY COHOMOLOGY OPERATION

BY

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1. Introduction

In 1926, J. W. Alexander [3] showed that the homology groups of X determined the homology groups of $S^N - X$ whenever X was embedded in S^N. In [4], Peterson showed how the stable primary cohomology operations in X determine those of $S^N - X$. In this paper, we show how the stable secondary cohomology operations in X determine those of $S^N - X$.

Heuristically, if Φ is a secondary cohomology operation defined on the kernel of θ with values in the cokernel of θ' (i.e., corresponding to the relation $\theta'\theta = 0$), then the dual of Φ will be a secondary cohomology operation defined on the kernel of $\chi(\theta')$ with values in the cokernel of $\chi(\theta)$ (i.e., corresponding to the relation $\chi(\theta)\chi(\theta') = \chi(\theta'\theta) = 0$), where χ is the involution in the Steenrod algebra (see [4]). If Φ is nonzero in X, then the dual of Φ will be nonzero in $S^N - X$. As in [4], this will be used to prove "nonembedding theorems."

We will state and prove our theorem in the language of J. F. Adams ([1] or [2]). One of the key steps in the proof is the fact that the two formulas in [5] are "dual" to each other.

2. Secondary cohomology operations

In this section, we recall Adams's definition of stable secondary cohomology operations with coefficients Z_p, [2].

Let (d, z, m) be such that $d:C_1 \to C_0$, $z \in C_1$, $d(z) = 0$, where C_1 and C_0 are free graded modules over the Steenrod algebra A. Let c_0 and c_r, $r = 1, \cdots, R$, be bases for C_0 and C_1 respectively, and let $\deg c_0 = 0$, $\deg c_r = q(r)$, $\deg d = 0$, and $\deg z = n + 1$. Let

$$\varepsilon: C_0 \to H^*(X) \qquad (= \textstyle\sum_{q>0} H^q(X; Z_p))$$

be a map of degree m. Then Φ is a secondary cohomology operation associated with (d, z, m) if it satisfies the following four axioms:

1. $\Phi(\varepsilon)$ is defined if $\varepsilon d = 0$.
2. $\Phi(\varepsilon) \in H^{m+n}(X)/\sum_{r=1}^{R} \alpha_r H^{m+q(r)-1}(X)$, where $z = \sum_{r=1}^{R} \alpha_r c_r$.
3. If $f:X \to Y$, and $\varepsilon: C_0 \to H^*(Y)$, then
$$\Phi(f^*\varepsilon) = f^*\Phi(\varepsilon) \in H^{m+n}(X)/\textstyle\sum_{r=1}^{R} \alpha_r H^{m+q(r)-1}(X).$$
4. Let (X, Y) be a pair, and let $i: Y \to X$, and $j: X \to (X, Y)$ be the

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