ON ALBANESE VARIETIES

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Let V be an r-dimensional algebraic variety, $(P(V), \Phi_0)$, $(A(V), \Psi_0)$ the Picard variety and the Albanese variety attached to V, Φ_0, Ψ_0 being the canonical homomorphisms of $\mathfrak{G}_a^{r-1}$, $\mathfrak{G}_a^0$ onto $P(V)$, $A(V)$ respectively, where $\mathfrak{G}_a^{r-1}$, $\mathfrak{G}_a^0$ denote the groups of divisors and of zero-cycles on V, respectively, which are algebraically equivalent to 0. $(P(V), \Phi_0)$ can be characterized as follows: Let G be any group variety, and Φ any "*algebraic*" group homomorphism of $\mathfrak{G}_a^{r-1}$ into G. Then there exists a (rational) homomorphism $\phi: P(V) \rightarrow G$ such that $\Phi = \phi \circ \Phi_0$. (We shall not dwell here upon the meaning of the word "algebraic"; it has a certain algebraic-geometrical sense, which will be fully explained on another occasion.[1]) Likewise $(A(V), \Psi_0)$ can be characterized in a similiar way: One has only to replace $\mathfrak{G}_a^{r-1}$ by $\mathfrak{G}_a^0$ in the above characterization of $(P(V), \Phi_0)$. Now it is known that (1) the kernel of Φ_0 is the group of divisors on V which are linearly equivalent to 0, (2) the rational mapping F of a total maximal Chow variety W of positive divisors on V onto $P(V)$ induced by Φ_0 is a regular mapping,[2] and (3) if W is a complete total Chow variety, the inverse image of a point of $P(V)$ by F is a Chow variety associated with a complete linear system. We shall prove in the present paper that $(A(V), \Psi_0)$ has properties corresponding to these properties of $(P(V), \Phi_0)$. Namely we shall prove that (1) the kernel of Ψ_0 is the group of zero-cycles on V which are regularly equivalent to 0,[3] (2) n being sufficiently large,[4] the rational mapping F_n of a Chow variety $V(n)$ of positive zero-cycles of degree n onto $A(V)$ induced by Ψ_0 is a regular mapping, and (3) n being again sufficiently large,[5] the inverse image $X_v^{(n)}$ of a point v of $A(V)$ by F_n is a regular variety.[6]

It was proved recently by Y. Taniyama [7] and A. Mattuck [6] that $X_v^{(n)}$ is irreducible. Our result gives additional information on $X_v^{(n)}$. Throughout

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[1] See a forthcoming paper of the author.

[2] Let f be a rational mapping of V into U, and k a common field of definition for V, U, and f. We shall say that f is a *regular mapping* if $k(x)$ is a regular extension of $k(f(x))$, where x is a generic point of V over k.

[3] As to the definition of "*regularly equivalent*", see below, §4.

[4] There is an example of V, such that the canonical mapping of $V = V(1)$ into $A(V)$ is purely inseparable, or separably algebraic, so that we must necessarily consider $V(n)$ for sufficiently large n to obtain an analogy of (2).

[5] T. Matsusaka communicated to me an example of V, such that $X_v^{(1)}$ is irreducible, but is not a regular variety, so that we must again consider $V(n)$ for large n.

[6] A variety V, such that there are no rational mappings from V into abelian varieties other than constant mappings, is called a *regular variety*. (This terminology is of the Italian school.)