HOMOLOGY OPERATIONS AND LOOP SPACES

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I. Introduction

Kudo and Araki [4] have computed the homology ring with coefficients integers mod 2, of the iterated loop spaces of an n-sphere. Their technique involved the definition of homology operations in so-called H_n-spaces. We give here a new treatment of these homology operations which leads us to a new operation of two variables, defined for all coefficient domains. This is done in Sections II and III. In Section IV we apply results of II and III to calculate the homology ring of the iterated loop spaces of iterated suspensions of a space mod 2, in terms of the homology of the original space mod 2, with the number of loop spaces less than or equal to the number of suspensions. The cohomology ring is also computed mod 2, if the number of loop spaces is less than the number of suspensions.

Some of the results of II and III may be applied to coefficients other than the integers mod 2. This will be done elsewhere [8].

The definition of loop space employed will be that of Moore (see [2], 22).

I would like to express my warm appreciation to Professor J. C. Moore. This paper is part of a dissertation written under his direction, presented to Princeton University.

II. H_n-spaces

In Sections II and III we reformulate the results of Kudo and Araki [4] in such a way that the techniques of Steenrod for defining cohomology operations [6] can be applied to obtain homology operations in H_n-spaces. In the course of this, besides the operations of one variable mod 2 of Kudo and Araki, we get a new operation of two variables which is defined for any coefficient domain.

Let π = the symmetric group on two letters. Then if X is any space, π acts on $X \times X$ by permuting the two coordinates. The group π also acts on the n-sphere S^n by the antipodal map. If π acts on two spaces M and N, let π act on $M \times N$ by $T(x, y) = (Tx, Ty)$, for $T \in \pi$.

DEFINITION. A space X is called an H_n-space if there exists an *equivariant* map

$$\phi: S^n \times (X \times X) \to X, \tag{1}$$

where π acts trivially on the right, such that there is an element $e \in X$ such

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