# ON COVERING DIMENSION AND INVERSE LIMITS OF COMPACT SPACES

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In 1937 H. Freudenthal proved that every metrizable compact space $X$ is homeomorphic with the inverse limit of an inverse sequence of compact polyhedra $P_i$, whose dimension $\dim P_i \leqq \dim X$ ([3], Satz 1, p. 229).[1] In this paper we ourselves propose to generalize Freudenthal's theorem to the case of Hausdorff compact spaces. Throughout the paper dimension is taken in the sense of finite open coverings and is denoted by dim.

It is well known that Hausdorff compact spaces can be characterized as inverse limits of inverse systems (over general directed sets) of compact polyhedra (see [2], Theorem 10.1, p. 284). This fact, together with Freudenthal's theorem, leads naturally to the conjecture that every compact Hausdorff space $X$ is homeomorphic with the limit of an inverse system of compact polyhedra $P_\alpha$, subjected to the additional requirement $\dim P_\alpha \leqq \dim X$. However, this conjecture is shown false in Section 5 of this paper, where we produce examples of 1-dimensional Hausdorff compact spaces which are not expressible as limits of polyhedra $P_\alpha$ with $\dim P_\alpha \leqq 1$.

Nevertheless, in Section 3 we show that every Hausdorff compact space $X$ is an inverse limit of metrizable compacta $X_\alpha$ with $\dim X_\alpha \leqq \dim X$ (Theorem 1). Combining this result with the theorem of Freudenthal we conclude that every Hausdorff compact space $X$ is a double iterated inverse limit of polyhedra $P_{\alpha i}$, satisfying $\dim P_{\alpha i} \leqq \dim X$.

Section 4 is devoted to another generalization of Freudenthal's theorem. This time we prove that every nonmetrizable Hausdorff compact space $X$ can be obtained as the inverse limit of a well-ordered system of Hausdorff compact spaces $X_\alpha$, where $\dim X_\alpha \leqq \dim X$, and in addition the weight[2] $w(X_\alpha)$ of every $X_\alpha$ is strictly smaller than the weight $w(X)$ of $X$ (Theorem 3).

The proofs of Theorems 1 and 3 depend on establishing the existence of a factorization of mappings $f:X \rightarrow Y$ through a compact space $Q$, satisfying $\dim Q \leqq \dim X$, $w(Q) \leqq w(Y)$. The first results of this kind are proved in Section 2 (Lemmas 3 and 4); the question is resumed in Section 3. From one of our factorization theorems follows a recent result of E. Sklyarenko on the compactification of normal spaces (Corollary 3).

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[1] Actually, Freudenthal proved a stronger statement, giving additional information concerning the nature of the bonding maps that appear in the sequence. In particular, all the maps can be assumed to be onto.

[2] The definition of weight is given in Section 1.