ON A CLASS OF DOUBLY TRANSITIVE PERMUTATION GROUPS[1]

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1. Introduction

In this paper we will study permutation groups satisfying the following conditions.

HYPOTHESIS I. *G is a doubly transitive permutation group on $m + 1$ letters in which no nontrivial permutation leaves three letters fixed.*

All known examples of permutation groups G satisfying Hypothesis I either contain a normal subgroup of order $m + 1$ or are contained in an exactly[2] triply transitive permutation group G_0, with $[G_0:G] \leqq 2$. In the latter case it is known [10] that $m = p^e$ for some prime p and that the Sylow p-groups of G are abelian. In view of this it seems reasonable to conjecture that the only permutation groups satisfying Hypothesis I are the ones just mentioned. In this paper we prove the following result which is a step in the direction of the conjecture.

THEOREM 1. *Let G be a permutation group of order $qm(m + 1)$ which satisfies Hypothesis I. Then either G contains a normal subgroup of order $m + 1$, or $m = p^e$ for some prime p. In the latter case, $[S_p:S_p'] < 4q^2$, where S_p is the Sylow p-group of G, and if $S_p' = \{1\}$, there exists an exactly triply transitive permutation group G_0 containing G such that $[G_0:G] \leqq 2$.*

Section 2 is devoted to the proof of Theorem 2 which is the main result of this paper. This theorem enables one to compute a large part of the character table for groups G which contain a subgroup M satisfying certain conditions (Hypothesis II in Section 2). The proof of Theorem 2 uses the fundamental result recently proved by J. G. Thompson [9], which together with the results of [5] and [7] show that the regular subgroup of a Frobenius group[3] is nilpotent. The special case of Theorem 2 in which the subgroup M is abelian was proved by R. Brauer and M. Suzuki [8] and has turned out to be a powerful tool in the study of finite linear groups[4] (see for example [3], [8]). Since

Received December 8, 1958.
[1] This research was supported by the United States Army and by the National Science Foundation.

[2] An exactly k-tuply transitive permutation group is a k-tuply transitive permutation group in which only the identity element leaves k or more letters fixed.

[3] By a Frobenius group is meant a group which contains a proper normal subgroup M, called the regular subgroup, with the property that no element in $M - \{1\}$ commutes with any element not in M. Elementary properties of such groups can be found in [5, Section 2].

[4] I am indebted to the authors of [3] for allowing me to see a copy of their manuscript before publication.